AF 61 (052) — 905
SR 10

OCTOBER 1970

AD 721 473

SCIENTIFIC REPORT No. 10

INFLUENCE OF IN-PLANE BOUNDARY CONDITIONS ON BUCKLING OF RING-STIFFENED CYLINDRICAL SHELLS

BY

TANCHUM WELLER
MENAHEM BARUCH
JOSEF SINGER

Department of Aeronautical Engineering
Technion — Israel Institute of Technology,
Haifa, Israel

TAE REPORT No. 101

61

AF 61 (052) - 905

SR 10

OCTOBER 1970

SCIENTIFIC REPORT No. 10

INFLUENCE OF IN-PLANE BOUNDARY CONDITIONS ON BUCKLING OF RING STIFFENED CYLINDRICAL SHELLS.

by

TANCHUM WELLER
MENAHEM BARUCH
JOSEF SINGER

Department of Aeronautical Engineering
Technion - Israel Institute of Technology,
Haifa, Israel

TAE REPORT No. 101

This document has been approved for public release and sale; its distribution is unlimited.

The research reported in this document has been sponsored by the Air Force Office of Scientific Research, through the European Office of Aerospace Research, United States Air Force under Contract AF61(052)-905 and Contract F61052-69-C0040.

TABLE OF CONTENTS

ABSTRACT

The effect of in-plane boundary conditions on the buckling loads of simply supported ring-stiffed cylindrical shells is studied. As in the case of unstiffened shells, the "weak" in-plane boundary conditions SS1 and SS2 yield here critical loads about one half of the "classical" loads. It was observed that the SS1 critical loads are identical with the SS2 loads and the SS4 loads are almost the same as the "classical" SS3 loads.

The combined effect of stiffener parameters and in-plane boundary conditions is studied. For internally stiffened shells the influence of in-plane boundary conditions is found to diminish with increasing values of stiffener eccentricity and area. No such effect is observed for externally stiffened shells. The buckling modes are also studied and found that they are dependent upon shell length (or Z) and upon stiffener location and parameters.

LIST OF SYMBOLS

A, B	- coefficients of additional displacements - Eqs. (9)
A_n, B_n, C_n	- coefficients of displacements - Eqs. (6)
A_{jn}, B_{jn}	- coefficients of additional displacements - Eqs. (13)
$A_{1n} \ldots A_{4n}$	- coefficients of additional displacements - Eqs. (14)
$A(n,m)$, $B(n,m)$, $D(n,m)$	- defined by Eqs. (23)
$(A_{o1})_{Ax}$, $(A_{o2})_{Ax}$	- coefficients of axisymmetric displacement - Eqs.(25)
A_2	- cross sectional area of ring
a	- distance between rings (see Fig. 1)
a_n, b_n	- defined by Eqs. (7)
D	- $Eh^3/12(1-\nu^2)$
D_{on}, D_{1n}, D_{2n}	- defined by Eqs. (7)
E	- moduli of elasticity
e_2	- distance between centroid of stiffener cross-section and middle surface of shell, positive when inside (see Fig. 1)
G	- shear moduli
h	- thickness of shell
I_{22}	- moment of inertia of ring cross-section about its centroidal axis
I_{o2}	- moment of inertia of ring cross-section about the middle surface of the shell.
I_{t2}	- torsion constant of ring cross-section

k	- $(1 + \mu_2)$
L	- length of shell between bulkheads
ℓ	- $(L/2)$
$M_x, M_\phi, M_{x\phi}, M_{\phi x}$	- moment resultants acting on element - Eqs. (2)
m	- integer
N_x, N , N_x ,	- membrane forces resultants acting on element
n	- integer, also number of half axial waves
P	- axial load
$Q(n,t)$	- defined by Eqs. (23)
R	- radius of shell
$R(m)$	- defined by Eqs. (23)
$S(m)$	- defined by Eqs. (23)
$T(n)$	- defined by Eqs. (23)
t	- number of circumferential waves
$U(n,t)$	- uefined by Eqs. (23)
u^*, v^*, w^*	- displacements (see Fig. 1)
u, v, w	- non-dimensional displacements ($= u^*/R$; v^*/R; w^*/R respectively)
$u_o(x)$, $v_o(x)$, $w_o(x)$	- additional displacements - Eqs. (6)
$(u_o)_n, (v_o)_n$	- displacements defined by Eqs. (9)
x^*, y^*, z^*, ϕ	- coordinates (See Fig. 1)
x, y, z	- non-dimensional coordinates ($= x^*/R$; y^*/R; z^*/R)
Z	- $(1-\nu^2)^{1/2}(L/R)^2(R/h)$

α	- exponents - Eqs. (9)
$\alpha_1 \cdots\cdots \alpha_4$	- roots of characteristic equation - defined by Eqs. (12)
β	- $\pi R/L$
δ	- Kronecker delta
ζ_2	- $(E_2 A_2 e_2\ R/aD)$
η_{o2}	- $(E_2 I_{o2}/aD)$
η_{t2}	- $(G_2 I_{t2}/aD)$
θ_j	- coefficients defined by Eq. (14)
λ	- $(PR/\pi D)$
λ_p	- $(R^3/D)p$
μ_2	- $(1-\nu^2)(E_2 A_2/E_{ah})$
χ_2	- $(1-\nu^2)(E_2 A_2 e_2/E_{ah} R)$

Subscripts following a comma indicate differentiation.

LIST OF TABLES.

LIST OF FIGURES

I. INTRODUCTION

The influence of the in-plane boundary conditions on the buckling load of unstiffened cylindrical shells under axial compression or longitudinal shear has been the subject of many studies [1 - 14]. For simply supported end conditions, the most significant result obtained was that the zero shear stress boundary conditions, SS1 ($N_x = N_{x\phi} = 0$) and SS2 ($u = N_{x\phi} = 0$), reduce the critical load to one half of the "classical" load - that obtained for the SS3 ($v = N_x = 0$) boundary conditions, while for the SS4 ($u = v = 0$) boundary conditions critical loads equal to the "Classical" ones were obtained. For unstiffened conical shells the effect was studied in [15], [16] and [17].

The influence of the in-plane end constraints on the buckling under lateral and hydrostatic pressure of unstiffened cylindrical shells was investigated in [18] and [19]. The effect in the case of free vibrations was also studied in [20] and [21]. The effect of prebuckling deformations on the buckling load of unstiffened shells was shown by different investigators [9, 20, 23] to be small except for very short shells. For stiffened shells this effect was found to be even smaller [24] and [25].

For stiffened-cylindrical shells the effect of the in-plane boundary conditions on the buckling load was studied in Refs. [26] and [27]. Axisymmetric buckling modes were not examined in these studies. Since, however,

earlier studies [28] showed that in the case of externally ring-stiffened cylindrical shells under axial compression these modes are the critical buckling modes, they should also be considered.

In the present report a different approach is employed to study the influence of the in-plane boundary conditions on the buckling under axial compression and hydrostatic pressure of simply supported ring-stiffened shells. The displacement method employed for conical shells in [29] and [15] and for cylindrical shells in [14] (which is an extension of that used earlier for classical boundary conditions [30])is applied. In the analysis, a solution is assumed for the w displacement, which solves the first two equilibrium equations exactly. The solution yields four constants which are determined by compliance with the appropriate boundary conditions (SS1; 2; 3 and 4) Then the third equation is solved by a standard Galerkin procedure. Axisymmetric buckling modes are included and an extensive parametric study is carried out.

It may be noted that the stiffening is assumed to be closely spaced and hence the stiffeners are taken as "smeared" or "distributed" over the entire shell, which implies that discreteness effects - usually negligible [31] and [32] - are not considered. The effects of eccentricity of loading are also not included in the present study.

2. EQUATIONS AND BOUNDARY CONDITIONS

The analysis is based on the stability equation of [30] for buckling under combined axial compression and hydrostatic pressure. For ring-stiffened cylindrical shells these equations become:

$$Eh/(1-\nu^2)[u_{,xx} + (\frac{1-\nu}{2})u_{,\phi\phi} + (\frac{1+\nu}{2})v_{,x\phi} - \nu w_{,x}] = 0 \quad (1a)$$

$$Eh/(1-\nu^2)[(\frac{1+\nu}{2})u_{,x\phi} + (1+\mu_2)v_{,\phi\phi} + (\frac{1-\nu}{2})v_{,xx} - (1+\mu_2)w_{,\phi} - \chi_2 w_{,\phi\phi\phi}] = 0 \quad (1b)$$

$$-(D/R)\{\zeta_2(2w_{,\phi\phi} - {}_{,\phi\phi\phi}) + w_{,xxxx} + (2+\eta_{t2})w_{,xx\phi\phi} + (1+\eta_{o2})w_{,\phi\phi\phi\phi}$$
$$+ 12(R/h)^2[(1+\mu_2)(w-v_{,\phi}) - \nu u_{,x}] + \lambda(\frac{w_{,xx}}{2}) +$$
$$+ \lambda_p[(\frac{w_{,xx}}{2}) + w_{,\phi\phi}]\} = 0 \quad (1c)$$

and the forces and moments acting on an element are given by:

$$N_x = Eh/(1-\nu^2)[u_{,x} + \nu(v_{,\phi} - w]$$
$$N_\phi = Eh/(1-\nu^2)[(1+\mu_2)(v_{,\phi} - w) + \nu u_{,x} - \chi_2 w_{,\phi\phi}]$$
$$N_{x\phi} = N_{\phi x} = Eh/2(1+\nu)[u_{,\phi} + v_{,x}]$$
$$(2)$$
$$M_x = -(D/R)[w_{,xx} + \nu w_{,\phi\phi}]$$
$$M_\phi = -(D/R)[w_{,\phi\phi}(1 + \eta_{o2}) + \nu w_{,xx} - \zeta_2(v_{,\phi} - w)]$$
$$M_{x\phi} = (D/R)(1-\nu)w_{,x\phi}$$
$$M_{\phi x} = -(D/R)[(1-\nu) + \eta_{t2}]w_{,x\phi}$$

In the case of simply supported shells the usual out of plane boundary conditions to be satisfied at the edges are:

$$w = M_x = 0 \quad \text{at} \quad \begin{array}{ll} x = -(\ell/R) & \text{or} \quad y = 0 \\ x = (\ell/R) & \text{or} \quad y = (\pi/\beta) \end{array} \tag{3}$$

The displacements u, v and w, which are the solutions of Eqs. (1), must also satisfy the appropriate in-plane boundary conditions, one of the following 4 sets,

$$\begin{array}{lll} \text{SS1;} & N_x = N_{x\phi} = 0 & \\ \text{SS2:} & u = N_{x\phi} = 0 & \text{at} \quad x = -(\ell/R) \quad \text{or} \quad y = 0 \\ \text{"Classical"SS3;} & v = N_x = 0 & \qquad x = (\ell/R) \quad \text{or} \quad y = (\pi/\beta) \\ \text{SS4;} & u = v = 0 & \end{array} \tag{4}$$

Since $w = 0$ at the edges also $w_{,\phi\phi} = 0$ at the boundaries and the condition for $M_x = 0$ at the boundaries can be replaced by

$$w = w_{,xx} = 0 \qquad \text{at} \quad \begin{array}{ll} x = -(\ell/R) & \text{or} \ y = 0 \\ x = (\ell/R) & \text{or} \ y = (\pi/\beta) \end{array} \tag{5}$$

On the basis of [14], [28] and [30] the displacements for the solution of Eqs. (1) are chosen as

$$u = [u_o(x) + \sum_{n=1}^{\infty} A_n \cos(n\beta y)] \sin(t\phi)$$
$$v = [v_o(x) + \sum_{n=1}^{\infty} B_n \sin(n\beta y)] \cos(t\phi) \qquad (6)$$
$$w = [w_o(x) + \sum_{n=1}^{\infty} C_n \sin(n\beta y)] \sin(t\phi)$$

The Fourier series terms of the displacements are solutions of Eqs. (1), as was shown in [28] and [30], and their substitution in Eqs. (1) yield the coefficients A_n and B_n in terms of C_n as in Eq. (16) of [30].

$$a_n = \frac{A_n}{C_n} = \frac{D_{1n}}{D_{on}}$$
$$b_n = \frac{B_n}{C_n} = \frac{D_{2n}}{D_{on}} \qquad (7)$$

where

$$D_{1n} = -(\frac{1+\nu}{2})\chi_2\, n\beta t^4 + (1 + \mu_2)(\frac{1-\nu}{2})n\beta t^2 - \nu(\frac{1-\nu}{2})n^3\beta^3$$

$$D_{2n} = (\frac{1-\nu}{2})\chi_2 t^5 + [\chi_2 n^2\beta^2 - (\frac{1-\nu}{2})(1 + \mu_2)]t^3 + $$
$$+[(\frac{1+\nu}{2})\nu - (1 + \mu_2)]n^2\beta^2 t$$

$$D_{on} = (\frac{1-\nu}{2})(1 + \mu_2)t^4 + [(1 + \mu_2) - \nu]\, n^2\beta^2 t^2 + (\frac{1-\nu}{2})n^4\beta^4$$

The additional displacement $w_o(x)$ is arbitrarily assumed to be equal to zero since the Fourier series terms fulfil the boundary conditions Eqs. (5). Substitution of $w_o(x) = 0$ in the first two stability equations Eqs. (1a) and (1b) yields two homogeneous differential equations for the additional displacements u_o and v_o.

$$u_{o,xx} + \left(\frac{1-\nu}{2}\right)u_{o,\phi\phi} + \left(\frac{1+\nu}{2}\right) v_{o,x\phi} = 0$$

$$\left(\frac{1+\nu}{2}\right)u_{o,x\phi} + (1+\mu_2)v_{o,\phi\phi} + \left(\frac{1-\nu}{2}\right)v_{o,xx} = 0 \tag{8}$$

The solution of Eqs. (8) can be written as

$$(u_o)_n = Ae^{\alpha x} \sin(t\phi)$$

$$(v_o)_n = Be^{\alpha x} \cos(t\phi) \tag{9}$$

Substitution of Eqs. (8) into (9) yields the following homogeneous algebraic equation

$$\begin{bmatrix} [\alpha^2 - (\frac{1-\nu}{2})t^2] & -(\frac{1+\nu}{2})\alpha t \\ (\frac{1+\nu}{2})\alpha t & [(\frac{1-\nu}{2})\alpha^2 - (1+\mu_2)t^2] \end{bmatrix} \times \begin{Bmatrix} A \\ B \end{Bmatrix} = 0 \tag{10}$$

and non vanishing values for A and B are obtained if the determinant of the coefficients of A and B is vanishing. Thus the characteristic equation for α is obtained

$$\left(\frac{1-\nu}{2}\right)\alpha^4 - t^2[(1+\mu_2) - \nu]\,\alpha^2 + t^4(1+\mu_2)\left(\frac{1-\nu}{2}\right) = 0 \tag{11}$$

and the roots of this equation are

$$\alpha_1 = -\alpha_2 = t\left[\frac{(k-\nu) + \sqrt{(k-1)(k-\nu^2)}}{(1-\nu)}\right]^{1/2}$$

$$\alpha_3 = -\alpha_4 = t\left[\frac{(k-\nu) - \sqrt{(k-1)(k-\nu^2)}}{(1-\nu)}\right]^{1/2} \tag{12}$$

where $\quad k = (1 + \mu_2)$

It can be shown that α_1 and α_2 are always real, because $(k-1)(k-\nu^2) > 0$. Similarly α_2 and α_4 are always real for all practical applications because

$$(k - \nu) > \sqrt{(k - 1)(k - \nu^2)} \text{ for a large range of } k.$$

With the above calculated values of α, the additional displacements u_o and v_o become

$$\begin{aligned}
(u_o)_n &= C_n \sin(t\phi) \sum_{j=1}^{4} A_{jn} \ell^{\alpha_j x} \\
(v_o)_n &= C_n \cos(t\phi) \sum_{j=1}^{4} B_{jn} \ell^{\alpha_j x}
\end{aligned} \tag{13}$$

From Eqs. (10) the relation between A_{jn} and B_{jn} is given by

$$B_{jn} = \theta_j A_{jn} \tag{14}$$

where $\quad \theta_j = \dfrac{2\alpha_j - (1 - \nu)t^2}{(1 + \nu)\alpha_j t}$

and $\quad \theta_1 = -\theta_2 \quad ; \quad \theta_3 = -\theta_4$

Hence the complete displacements can be written as

$$u = \sin(t\phi) \sum_{n=1}^{\infty} C_n [a_n \cos(n\beta y) + A_{1n} sh(\alpha_1 x) + A_{2n} sh(\alpha_3 x) + A_{3n} ch(\alpha_1 x) + A_{4n} ch(\alpha_3 x)]$$

$$v = \cos(t\phi) \sum_{n=1}^{\infty} C_n [b_n \sin(n\beta y) + \theta_1 A_{1n} ch(\alpha_1 x) + \theta_3 A_{2n} ch(\alpha_3 x) + \theta_1 A_{3n} sh(\alpha_1 x) + \theta_3 A_{4n} sh(\alpha_3 x)]$$

$$w = \sin(t\phi) \sum_{n=1}^{\infty} C_n \sin(n\beta y) \tag{15}$$

These equations include four constants of integration A_{jn} which will be determined from the appropriate in-plane boundary conditions.

3. COMPLIANCE WITH IN-PLANE BOUNDARY CONDITIONS

The values of the constants A_{jn} are determined by enforcing the four sets of boundary conditions listed in (4).

For example, in case SS4 the requirement

$$u = v = 0 \text{ at } x = -(\ell/R) \text{ or } y = 0$$
$$x = (\ell/R) \text{ or } y = (\pi/\beta)$$

yields

$$\begin{bmatrix} sh(\alpha_1\ell/R) & sh(\alpha_3\ell/R) & -ch(\alpha_1\ell/R) & -ch(\alpha_3\ell/R) \\ sh(\alpha_1\ell/R) & sh(\alpha_3\ell/R) & ch(\alpha_1\ell/R) & ch(\alpha_1\ell/R) \\ \theta_1 ch(\alpha_1\ell/R) & \theta_3 ch(\alpha_3\ell/R) & -\theta_1 sh(\alpha_1\ell/R) & -\theta_3 sh(\alpha_3\ell/R) \\ \theta_1 ch(\alpha_1\ell/R) & \theta_3 ch(\alpha_3\ell/R) & \theta_1 sh(\alpha_1\ell/R) & \theta_3 sh(\alpha_3\ell/R) \end{bmatrix} \begin{Bmatrix} A_{1n} \\ A_{2n} \\ A_{3n} \\ A_{4n} \end{Bmatrix} = \begin{Bmatrix} a_n \\ (-1)^n a_n \\ 0 \\ 0 \end{Bmatrix} \tag{16}$$

This matrix equation can be divided into two matrix equations one yielding symmetric modes of buckling - n =1, 3,5... and one antisymmetric modes - n = 2,4,6...

For symmetric modes: $n = 1, 3, 5\ldots$

$$A_{3n} = A_{4n} = 0$$

$$\begin{bmatrix} sh(\alpha_1\ell/R) & sh(\alpha_3\ell/R) \\ \theta_1 ch(\alpha_1\ell/R) & \theta_3 ch(\alpha_3\ell/R) \end{bmatrix} \begin{Bmatrix} A_{1n} \\ A_{2n} \end{Bmatrix} = \begin{Bmatrix} a_n \\ 0 \end{Bmatrix} \tag{17a}$$

For antisymmetric modes: $n = 2, 4, 6...$

$$A_{1n} = A_{2n} = 0$$

$$\begin{bmatrix} ch(\alpha_1 \ell/R) & ch(\alpha_3 \ell/R) \\ \theta_1 sh(\alpha_1 \ell/R) & \theta_3 sh(\alpha_3 \ell/R) \end{bmatrix} \begin{Bmatrix} A_{3n} \\ A_{4n} \end{Bmatrix} = \begin{Bmatrix} -a_n \\ 0 \end{Bmatrix} \tag{17b}$$

In case SS3

$$N_x = v = 0$$

In a similar manner to case SS4, a set of 4 homogereous equations is obtained for which only the trivial vanishing solution exists

$$A_{1n} = A_{2n} = A_{3n} = A_{4n} = 0 \tag{18}$$

and the "classical" solution of [28] is obtained.

In case SS 2

$$u = N_{x\phi} = 0$$

For symmetric modes: $n = 1, 3, 5$

$$A_{3n} = A_{4n} = 0$$

$$\begin{bmatrix} sh(\alpha_1 \ell/R) & sh(\alpha_3 \ell/R) \\ \alpha_1\theta_1 sh(\alpha_1 \ell/R) & \alpha_3\theta_3 sh(\alpha_3 \ell/R) \end{bmatrix} \begin{Bmatrix} A_{1n} \\ A_{2n} \end{Bmatrix} = \begin{Bmatrix} a_n \\ n\beta b_n \end{Bmatrix} \tag{19a}$$

antisymmetric
whereas for ~~asymmetric~~ modes: $n = 2,4,6\ldots$

$$\underline{A_{1n} = A_{2n} = 0}$$

and

$$\begin{bmatrix} ch(\alpha_1 \ell/R) & ch(\alpha_3 \ell/R) \\ \alpha_1\theta_1 ch(\alpha_1 \ell/R) & \alpha_3\theta_3 ch(\alpha_3 \ell/R) \end{bmatrix} \begin{Bmatrix} A_{3n} \\ A_{4n} \end{Bmatrix} = \begin{Bmatrix} -a_n \\ -n\beta b_n \end{Bmatrix} \qquad (19b)$$

In case <u>SS 1</u>

$$N_x = N_{x\phi} = 0$$

For symmetric modes: $n=1, 3, 5\ldots$

$$\underline{A_{3n} = A_{4n} = 0}$$

and

$$\begin{bmatrix} (\alpha_1 - \nu t\theta_1) ch(\alpha_1 \ell/R) & (\alpha_3 - \nu t\theta_3) ch(\alpha_3 \ell/R) \\ (t + \alpha_1\theta_1) sh(\alpha_1 \ell/R) & (t + \alpha_3\theta_3) sh(\alpha_3 \ell/R) \end{bmatrix} \begin{Bmatrix} A_{1n} \\ A_{2n} \end{Bmatrix} = \begin{Bmatrix} 0 \\ ta_n + n\beta b_n \end{Bmatrix} \qquad (20a)$$

antisymmetric
whereas for ~~asymmetric~~ modes: $n = 2,4,6\ldots$

$$\underline{A_{1n} = A_{2n} = 0}$$

and

$$\begin{bmatrix} (\alpha_1 - \nu t\theta_1) sh(\alpha_1 \ell/R) & (\alpha_3 - \nu t\theta_3) sh(\alpha_3 \ell/R) \\ (t + \alpha_1\theta_1) ch(\alpha_1 \ell/R) & (t + \alpha_3\theta_3) ch(\alpha_3 \ell/R) \end{bmatrix} \begin{Bmatrix} A_{3n} \\ A_{4n} \end{Bmatrix} = \begin{Bmatrix} 0 \\ -ta_n - n\beta b_n \end{Bmatrix} \qquad (20b)$$

4. SOLUTION

Now every term of the displacements series Eqs. (15) is a solution of the first two stability equations (1a) and (1b).

The third equation (1c) is solved by a Galerkin procedure.

$$\int_{o}^{2\pi} d\phi \int_{(-\ell/R);0}^{(\ell/R);(\pi/\beta)} -(D/R)\{\zeta_2(2w_{,\phi\phi}-v_{,\phi\phi\phi})+w_{,xxxx}+(2+\eta_{t1})w_{,xx\phi\phi}+(1+\eta_{o2})w_{,\phi\phi\phi\phi}$$

$$+12(R/h)^2[(1+\mu_2)(w-v_{,\phi})-\nu u_{,x}]+\lambda(\frac{w_{,xx}}{2})+\lambda_p[(\frac{w_{,xx}}{2}+w_{,\phi\phi}]\}$$

$$\cdot\, w_m dx = 0 \qquad (21)$$

Substitution of Eqs. (15) and performing the necessary integration yields a system of linear homogeneous algebraic equations for the unknowns λ and λ_p which are the critical load parameters

$$\sum_{n}^{\infty} C_n[A(n,m) + \lambda B(n,m) + \lambda_p D(n,m)] = 0 \qquad (22)$$

$$m = 1,2,3,\ldots$$

where m; n = 1,3, 5... for symmetric modes of buckling and

m; n = 2,4, 6... for asymmetric modes of buckling

and where for the symmetric modes

$$A(n,m) = \delta_{mn}\frac{\pi^2}{2\beta}Q(n,t) + ch(\alpha_1\ell/R)R(m)A_{1n} + ch(\alpha_3\ell/R)S(m)A_{2n} \qquad (23a)$$

$$B(n,m) = \delta_{mn}T(n) \qquad (23b)$$

$$D(n,m) = \delta_{mn}U(n,t) \qquad (23c)$$

and for the ~~asymmetric~~ antisymmetric modes

$$A(n,m) = \delta_{mn}\pi^2/2\beta\ Q(n,t) - sh(\alpha_1\ell/R)R(m)\ A_{3n} - sh(\alpha_3\ell/R)S(m)\ A_{4n} \tag{23d}$$

$$B(n,m) = \delta_{mn}T(n) \tag{23e}$$

$$D(n,m) = \delta_{mn}U(n,t) \tag{23f}$$

Q(n,t), R(m), S(m), T(n), U(n,t) in Eqs. (23) are defined as follows:

$$Q(n,t) = \{[\zeta_2 t^2 - 12(R/h)^2(1+\mu_2)]tb_n - 12(R/h)^2[\nu n\beta a_n + (1+\mu_2)] - n^4\beta^4$$

$$- t^2[t^2(1 + n_{o2}) + (2 + n_{t2})n^2\beta^2 - 2\zeta_2]\}$$

$$R(m) = \frac{2\theta_1 m\beta\pi}{\alpha_1^2 + m^2\beta^2}\ \{\zeta_2 t^3 - 12(R/h)^2[t(1 + \mu_2) - \nu(\frac{\alpha_1}{\theta_1})]\}$$

$$S(m) = \frac{2\theta_3 m\beta\pi}{\alpha_3^2 + m^2\beta^2}\ \{\zeta_2 t^3 - 12(R/h)^2[t(1 + \mu_2) - \nu(\frac{\alpha_3}{\theta_3})]\}$$

$$T(n) = (\pi^2/2\beta)\ \frac{n^2\beta^2}{2}$$

$$U(n,t) = (\pi^2/2\beta)[t^2 + \frac{n^2\beta^2}{2}]$$

Truncating of the displacement series at n = N yields a NxN stability matrix Eq. (22) whose lowest eigenvalue yields the critical load for n = N. In the calculations, the circumferential number of waves (t) is treated as a parameter for which the minimal critical value is found for a given n. The size NxN of the matrix is determined by the convergence criterion for the critical load.

5. AXISYMMETRIC BUCKLING.

For this mode of buckling $t = 0$ and there is no dependence on v and θ and the equilibrium equation (1) degenerate to

$$(Eh/1-\nu^2)[u_{,xx} - \nu w_{,x}]=0 \tag{24a}$$

$$-(D/R)[-w_{,xxxx}+ 12(R/h)^2(\nu u_{,x} - w) - (\lambda + \lambda_p)(\frac{w_{,xx}}{2})] = 0 \tag{24b}$$

If as before, $w_o = 0$, the additional displacement u_o becomes

$$u_o = (A_{o1})_{Ax} X + (A_{o2})_{Ax} \tag{25}$$

and the complete displacements are then

$$u = \sum_{n=1}^{\infty} C_n[a_n \cos(n\beta y) + (A_{o1})_{Ax}X + (A_{o2})_{Ax}]$$

$$w = \sum_{n=1}^{\infty} C_n \sin(n\beta y) \tag{26}$$

and, as before, the coefficients are determined by compliance with the appropriate boundary conditions.

For axisymmetric buckling the in-plane boundary conditions (4) are given by:

In cases S.S.1 and S.S.3: $N_x = 0$ at $x = -(\ell/R)$ or $y = 0$

and cases S.S.2 and S.S.4: $u = 0$ $x = (\ell/R)$ or $y = (\pi/\beta)$ (27)

The coefficients are therefore:

For S.S.4 and S.S.2

$$(A_{o1})_{Ax} = \begin{cases} \frac{2\beta}{\pi} a_n & n = 1, 3, 5\ldots \\ 0 & n = 2, 4, 6\ldots \end{cases}$$

$$(A_{o2})_{Ax} = \begin{cases} 0 & n = 1.3.5\ldots \\ -a_n & n = 2.4.6\ldots \end{cases} \qquad (28)$$

and for SS3 and SS1

$$(A_{o1})_{Ax} = (A_{o2})_{Ax} = 0 \qquad (29)$$

$(A_{o2})_{Ax}$ is assumed to vanish because it represents a rigid body translation in the axial direction and thus can be ignored:

The second stability equation (24b) is again solved by the Galerkin procedure:

$$\int_{o}^{2\pi} d\phi \int_{-\ell/R;\ o}^{\ell/R;\ (\pi/\beta)} (D/R)[w_{,xxxx} + 12(R/h)^2(1+\mu_2)(\nu u_{,x}-w)+\frac{w_{,xx}}{2}(\lambda+\lambda_p)]w_m dx = 0 \qquad (30)$$

which yields for the symmetric modes n = 1,3,5... the set of linear algebraic homogeneous equation

$$C_n\{(-\pi^2/2\beta)\delta_{mn}\{24(R/h)^2[a_n\ n\beta\nu+(1+\mu_2)]+2\ n^4\beta^4]\}+(48/m\beta)(A_{o1})_{Ax}(R/h)^2\pi\nu +$$

$$+ n^2\beta^2(\pi^2/2\beta)\ \delta_{mn}(\lambda + \lambda_p)\} = 0 \qquad m = 1,3,5,\ldots. \qquad (31)$$

and for the asymmetric modes - n = 2, 4,6.. another set

$$\delta_{mn}C_n(\pi^2/2\beta)\{24(R/h)^2[a_n\ n\beta\nu + (1+\mu_2)] + 2n^4\beta^4 + n^2\beta^2(\lambda+\lambda_p)\} = 0 \qquad (32)$$

$$m = 2,4,6,\ldots.$$

6. NUMERICAL RESULTS AND DISCUSSION

In the numerical work two main stiffener configurations, differing in their eccentricity, were studied, one with $(e_2/h) = \pm 1$, $(A_2/ah) = 0.5$, $(I_{22}/ah^3) = 2$ and $n_{t2} = 3$ (Table 1a) and the other one with $(e_2/h) = \pm 5$, $(A_2/ah) = 0.5$, $(I_{22}/ah^3) = 2$ and $n_{t2} = 3$ (Table 1b). In both studies the shell geometry was varied in the ranges $0.03 \leq (L/R) \leq 2.00$ and $100 \leq R/h \leq 2000$. The critical loads obtained for these geometries of shells and stiffeners are presented in Tables 1a and 1b. These loads are computed for the four simple support in-plane boundary conditions SS1 to SS4. The calculated critical loads are compared with the "classical" bucklings loads (SS3 boundary conditions) and the results are plotted as a function of the Batdorf parameter Z in Figs. 2a and 2b and as a function of the nondimensionalized shell length (L/R) in Fig. 3 with (R/h) as an additional parameter.

From Figs. 2a and 2b, as well as Fig. 3, it can be seen that the SS1 boundary conditions yield exactly the same critical loads as the SS2 boundary conditions and that the SS4 boundary conditions yield the same loads as the "classical" SS3 conditions. From Figs. 2a and 2b it can be seen that in the very low range of the Batdorf parameter $Z < 0.2$ the different in-plane boundary conditions do not differ the critical loads. But from Tables 1a and 1b one may also observe that, though the critical loads are identical the corresponding buckling modes are completely different.

The SS1 and SS2 boundary conditions yield asymmetric buckling modes whereas the SS3 and SS4 boundary conditions yield axisymmetric buckling modes.

It may be noted that for asymmetric modes the critical number of circumferential waves is $t = 2$ in many cases. For such small numbers of circumferential waves the Donnell type stability equations employed in the analysis are inaccurate in general. However, since the critical loads are found to increase only very slightly if t is increased from 2 to 4 or 5 (usually by less than 1%), the buckling loads with $t = 2$ can here be taken as close approximations.

With increasing values of Z up to values of $Z \approx 4.7$ the critical loads corresponding to SS1 and SS2 boundary conditions decrease to values as low as 35% of the "classical" critical loads for $Z \approx 4.7$. Beyond this value of Z, the SS1 and SS2 critical loads increase with Z, reaching values of about one half of the "Classical" load for $Z \approx 10$. For $Z > 10$ there is a slight increase of the SS1 and SS2 critical loads with increasing Z. This behavior differs slightly from that of unstiffened shells for which the SS1 and SS2 boundary conditions yield half the critical "classical" load independent of _ (See [6], [7], [13] and [14]).

Figures 2a and 2b show clearly that the influence of the in-plane boundary conditions depends on the eccentricity of the stiffeners. In Fig. 2a it is observed that in the case of low values of eccentricity, $(e_2/h) = \pm 1$ both externally or internally stiffened shells are equally affected by the in-plane boundary conditions for values of Z larger than 10. A noticeable

difference between external and internal stiffening appears there only in the range $4 < Z < 10$ where first external stiffening yields lower ratios of critical loads than internal stiffening and then the effect is inverted but of smaller magnitude.

With the larger values of eccentricity $(e_2/h) = \pm 5$ in Fig. 2b a noticeable difference is evident between internal and external stiffening for low value of Z $(Z > 1)$ as well as for larger values of Z. External stiffening yields lower ratios of SS1 and SS2 to SS3 loads for most shell geometries, except for a small region $6 < Z < 9$ where the effect is inverted. Similar conclusions can be drawn from Fig. 3.

There is some scatter of the points representing the ratios of the critical loads when plotted versus the shell geometry parameter Z. There are two reasons for this scatter. First the shell geometry parameter is only an approximate overall parameter in the case of stiffened shells and hence most of the scatter disappears when the ratios are plotted for the separate geometric parameters (See Fig. 3). Secondly the critical values occur at different circumferential curve numbers for different in-plane boundary conditions. The necessity of integral values causes the well known "ripples" in the curves of buckling load versus geometry parameters, which differ for each case and result in scatter when divided to give the ratios of the buckling loads.

In [28] it was concluded that externally ring-stiffened cylindrical shells should always buckle in an axisymmetric mode. The present results (Tables 1a and 1b) extend this conclusion to the SS 4 boundary conditions which are also shown to yield axisymmetric modes of buckling for such shells.

In Table 2 and Fig.4 the variation of the influence of the in-plane boundary conditions with increasing eccentricity is studied. The geometries of the shells are given in Table 2. From Fig. 4 it can be seen that for external stiffening the influence of in plane-boundary conditions does not change with magnitude of the eccentricity. For inside stiffening, on the other hand, the magnitude of the eccentricity affects the influence of the in-plane-boundary conditions which is reduced with increasing eccentricity.

The variation of the influence of the in-plane boundary conditions with increase of the rings area parameters (A_2/ah) is investigated in Table 3 and Fig. 5. Geometry and dimensions of the shells studied are presented in Table 3. From Fig. 5 it is seen that for externally stiffened shells the magnitude of the ring-area is practically inmaterial whereas for inside stiffening it is a major factor. The SS1 and SS2 critical loads increase noticeably with increasing values of the area parameter (A_2/ah) and approach the "classical"(SS3) critical load.

In Table 4, the effect of increasing the moment of inertia parameters (I_{22}/ah^3), in relation to the influence of the in-plane boundary conditions is studied on some of the shells. The dimensions and geometry of the shells examined are given in Table 4. The moment of inertia is found to have no

effect on the critical load ratios of the "weak" boundary conditions SS1 and SS2. In Table 5 and Fig. 6 the effect of increase of moment of inertia is also studied for a wider range of the parameter, $1 < I_{22}/ah^3 < 50$, with the same negative result. The dimensions and geometry of the shells are listed in Table 5.

The buckling modes of the w displacement (radially inwards) were also studied (Tables 1a and 1b). Some of the results are given in Figs. 7 to 11.

In Figs. 7a to 7d the buckling modes for "thick" shells with $(R/h)=100$ and low eccentricity $(e_2/h) = \pm 1$ are presented. It can be seen from these figures that short shells (Fig. 7a - $Z = .954$ and Fig. 7b $Z = 8.59$) yield identical modes for all the in-plane of boundary conditions (SS1 to SS4). Note also that the modes are identical for internal and external stiffening. As the length of shells increases (Fig. 7c - $Z = 95.4$ and Fig. 7d- $Z = 382$), the buckling modes for the SS1 and SS2 boundary conditions remain identical and independent or stiffener location. It can be seen that these "weak" boundary conditions are characterized by edge buckling which is the reason for their low critical loads. On the other hand the SS3 and SS4 boundary corditions yield modes that differ completely from those of SS1 and SS2 and depend also slightly on the stiffener location. Note that for internal stiffening the SS4 mode differs slightly from the SS3 mode while for external stiffening the modes are identical. Fig. 7d shows a more pronounced difference between the SS3 and SS4 modes with further increase of the shell length. The

SS3 mode now significantly differs from the SS4 mode, and internal and external stiffeners yield different modes of buckling.

In Figs. 8a to 8d the buckling modes are given for similar shells but with a larger ring-eccentricity- $(e_2/h) = \pm 5$. Increase in eccentricity changes the buckling modes. The SS3 and SS4 buckling modes are now not identical with the SS1 and SS2 modes, even for short shells: Fig. 8a- $Z = .954$ and Fig. 8b - $Z = 8.59$. For the short shell (Fig. 8a) no dependence upon the location of stiffeners is observed for all in-plane boundary conditions, whereas location dependence can be seen for the medium shell of Fig. 8b. For this shell, the SS3 mode differs for internal and external stiffeners. With increase of the shell length, the modes of Figs. 8c and 8d are obtained which exhibit a behavior similar to that in Figs. 7c and 7d.

In Figs. 9a to 9d the buckling modes are presented for "thin" shells, $(R/h) = 2000$, having a low value of eccentricity. The reduction of shell thickness, or rather the inrease in Z, yields different modes for the SS1 and SS2 boundary conditions than for SS3 and SS4. Note that for the short shell of Fig. 9a - $Z = 19.1$ the modes are location independent. With increase in length of shell (Fig. 9b - $Z = 477$)the SS3 modes begin to differ from the SS4 modes, and the SS4 modes are also dependent upon the location of stiffeners. From Figs. 9c and 9d it can be seen that with further increase in shell length the SS1 and SS2 boundary conditions yield identical buckling modes that are location independent. Similar conclusions apply to the SS3 and SS4 boundary conditions.

In Figs. 10a to 10d the buckling modes are studied for shells with the same geometry as Figs.9a to 9d, except for larger eccentricity, $(e_2/h) = \pm 5$. For short shells, the results of Fig. 10a - Z = 19.1 and Fig. 10b - Z = 477 are similar to those of Fig. 9a and 9b. With a further increase of shell length, a significant effect of the increased eccentricity is noted for SS4 modes of internally stiffened shells (Figs. 10c - Z = 1910 and 10d - Z = 7630). For corresponding externally stiffened shells no similar effect is observed.

The influence of stiffener eccentricity on the SS4 modes in the case of long shells is studied in Figs. 11a to 11c. A significant effect is only observed for large values of eccentricity (Figs. 11b - Z = 8600; $e_2/h = 5$ and 11c -z = 8600; $(e_2/h) = 5$). From Figs. 11b and 11c it can also be seen that an increase in the moment of inertia of the stiffeners changes the SS.4 modes noticeably.

It should be pointed out that in all the "long" shells the SS1 and SS2 modes are always characterized by edge buckling, which can explain the significant reduction in the critical loads corresponding to these boundary conditions. These boundary conditions yield buckling modes which were almost always one sided - inwards (positive w displacement), except for long shells with large eccentricities as in Figs. 11b and 11c.

7. CONCLUSIONS

a. As for unstiffened shells, the critical loads of ring-stiffened shells depend on the in-plane boundary conditions. The "weak" SS1 and SS2 condition yield identical buckling loads which are about one half the "classical" SS3 loads. The SS4 boundary conditions yield critical loads which are practically equal to the "classical" loads, or very slightly larger.

b. The buckling loads of very short shells - $Z < 0.1$ are independent of the in-plane boundary conditions.

c. In the range of $0.1 < Z < 10$ the SS1 and SS2 boundary conditions yield critical loads which are as low as 35% of the corresponding "classical" load.

d. For externally stiffened shells the influence of in-plane boundary conditions is not affected by the stiffener geometry.

e. For internally stiffened shell, on the other hand, critical loads for the "weak" in-plane boundary conditions SS1 and SS2 increase with stiffener area and eccentricity. For very large stiffener area they approach the "classical" SS3 critical loads. Changes in the moment of inertia of stiffeners have a negligible effect on the influence of in-plane boundary conditions.

f. The buckling modes depend on the shell length (or Z) and on the stiffeners geometry.

ACKNOWLEDGEMENT

The authors would like to thank Mr. Z. Yerushalmi for assistance with the programming and Mrs. A. Harari for assistance with the computations.

REFERENCES

1. Ohira, H., "Local Buckling Theory of Axially Compressed Cylinders" Proceedings of the Eleventh Japan National Congress for Applied Mechanics, 1961, pp. 37-41.

2. Singer, J., "The Effect of Axial Constraint on the Instability of Thin Cylindrical Shells Under Uniform Axial Compression", International Journal of Mechanical Sciences, Vol. 4, No. 2, May 1962, pp.253-258; Also TAE Report 16, Sept. 1961.

3. Ohira, H. "Linear Local Buckling Theory of Axially Compressed Cylinders and Various Eigenvalues" Proceedings, 5th International Symposium on Space Technology and Science, Tokyo, 1963, pp.511-526.

4. Hoff, N.J. and Rehfield L.A., "Buckling of Axially Compressed Cylindrical Shells at Stresses Smaller than the Classical Critical Value", Journal of Applied Mechanics, Vol. 32, No. 3, Trans. ASME, Vol. 87, Series E, Sept. 1965. pp. 542-546; also SUDAER Report 191, May 1964.

5. Hoff, N.J., "Low Buckling Stresses of Axially Compressed Circular Cylindrical Shells of Finite Length", Journal of Applied Mechanics, Vol. 32, No. 3, Trans. ASME, Series E. Vol. 87, Sept. 1965, pp. 533-541; also SUDAER Report 192, July 1964.

6. Hoff, N.J. and Soong, T.C., "Buckling of Circular Shells in Axial Compression," International Journal of Mechanical Sciences, Vol. 7, 1965, pp.489-520; also SUDAER Report 204, Aug. 1964.

7. Hoff, N.J., "The Effect of Edge Conditions on the Buckling of Thin-Walled Circular Cylindrical Shells in Axial Compression", Proc. of the 11th International Congress of Applied Mechanics, Munchen, 1964, Springer Verlag 1966, pp.326-331.

8. Thielemann, W., and Esslinger, M., "Einfluss der Randbedingungen auf die Beullast von Kreiszylinderschalen", Der Stahlbau,Vol.33,, No. 12, Dec. 1964. pp. 3-11.

9. Almroth, B.O., "Influence of Edge Conditions on the Stability of Axially Compressed Cylindrical Shells", AIAA Journal, Vol. 4, No. 1, Jan.1966, pp.134-140; Also NASA Contractor Report CR-161, Feb. 1965.

10. Ohira, H., "Local Buckling Theory for an Axially Compressed Circular Cylinder of Finite Length", Proc. of the 14th Japan National Congress for Applied Mechanics, 1964.

11. Fischer, G., "Influence of Boundary Conditions on Stability of Thin-Walled Cylindrical Shells under Axial Load and Internal Pressure", AIAA Journal, Vol. 3, No. 4, Apr. 1965, pp. 736-738.

12. Almroth, B.O., "Influence of Imperfections and Edge Restraint on the Buckling of Axially Compressed Cylinders", presented at the AIAA/ASME 7th Structure and Materials Conference, Cocoa Beach, Fla., April 18-20, 1966.

13. Kobayashi, S., "The influence of the Boundary Conditions on the Buckling Load of Cylindrical Shells under Axial Compression", NASA Contractor Report CR-558, Aug. 1966.

14. Baruch, M., and Harari, O., "Buckling of Cylindrical Shells under Longitudinal Shear Loads", TAE Report 81, Technion Research and Development Foundation," Haifa, Israel, November 1967.

15. Baruch, M., Harari, O. and Singer J., "Low Buckling Loads of Axially Compressed Conical Shells", J. App. Mech. paper No. 70-APM-AA.

16. Tani, J., and Yamaki, N., "Buckling of Truncated Conical Shells under Axial Compression" AIAA J. Vol. 8, No. 3, pp. 568-571, March 1970.

17. Singer, J., Barcuh, M., and Reichental, Y., " Influence of In-Plane Boundary Conditions on the Buckling of Clamped Conical Shells", to be published as TAE Report No. 93, Technion Research and Development Foundation, Haifa, Israel.

18. Sobel, L.H., "Effects of Boundary Conditions on the Stability of Cylinders Subject to Lateral and Axial Pressures", AIAA J., Vol. 2, No. 8, pp. 1437-1440, August 1964.

19. Yamaki, N., "Buckling of Circular Cylindrical Shells under External Pressure", Reports of the Inst. of High Speed Mechanics, THOKU University, Vol. 20, pp. 35-55, 1968/1969.

20. Forsberg, K. " Influence of Boundary Conditions on the Model Characteristics of Thin Cylindrical Shells", AIAA Journal, Vol. 2, No. 12,pp.2150-2157, December 1964.

21. Seggelke, P., - Schwingungsverhalten von axialsymmetrisch belasteten dünnenwandigen Kreiszylindern bei verchidenen Randbedingungen" Deutsche Luft-und Raumfahrt DLR FB 64-63, December 1964.

22. Yamaki, N., "Influence of Prebuckling Deformations on the Buckling of Circular Cylindrical Shells Under External Pressure", AIAA J., Vol. 7, No. 4, pp.753-755, April 1969.

23. Kobayashi, S., "The Influence of Prebuckling Deformations on the Buckling Load of Cylindrical Shells under Axial Compression", Journal of the Japan Society for Aeronautical and Space Sciences, Vol. 15, No. 163, pp.258-263, 1967/1969 (In Japanese with English Abstract).

24. Kobayashi, S. "The Influence of Prebuckling Deformations on the Buckling Load of Orthotropic Cylindrical Shells under Axial Compression", Transactions of the Japan Society for Aeronautical and Space Sciences, Vol. 11, No. 19, pp.60-68, 1968.

25. Singer, J., Arbocz, J. and Babcock, C.D. Jr., "Buckling of Imperfect Stiffened Cylindrical Shells under Axial Compression" presented at the 11th AIAA/ASME Structures, Structural Dynamics and Materials Conference, Denver,Colorado, April 1970.

26. Soong, T.C., "Influence of Boundary Constraints on the Buckling of Eccentrically Stiffened Orthotropic Cylinders", presented at the 7th International Symposium on Space Technology and Science, Tokyo, May, 1967.

27. Seggelke, P., and Geier, B., " Das Beulverhalten Versteifter Zylinderschalen", Zeitschrift fur Flugwissenschaften, Vol. 15, No. 12, pp.477-490, December 1967.

28. Singer, J., Baruch, M., and Harari, O., " On the Stability of Eccentrically Stiffened Cylindrical Shells under Axial Compression", Int. J. of Solids and Structures, Vol. 3, No. 4, pp.445-470, July 1967, Also TAE Report No. 44,Technion Research and Development Foundation, Haifa, Israel, December 1965.

29. Baruch, M., Harari, O., and Singer, J., "Effect of In-Plane Boundary Conditions on the Stability of Conical Shells Under Hydrostatic Pressure", Proceedings of the 9th Israel Annual Conf. on Aviation and Astronautics, Israel Journal of Technology, Vol. 5, No. 1, pp. 12-24, Feb. 1967.

30. Baruch, M., and Singer, J., "Effect of Eccentricity of Stiffeners on the General Instability of Stiffened Cylindrical Shells under Hydrostatic Pressure", J. of Mechanical Engineering Sciences, Vol. 5, No. 1. pp. 23 27, March 1963.

31. Singer, J., and Haftka, R., "Buckling of Discretely Ring Stiffened Cylindrical Shells" Proceedings of the 10th Israel Annual Conf. on Aviation and Astronautics, Israel Journal of Technology, Vol. 6, No. 1-2, pp.125-137, February 1968. Also TAE Report No. 67, Technion Research and Development Foundation, Haifa, Israel, August 1967.

32. Block, D., "Influence of Discrete Ring Stiffeners and Prebuckling deformations on the Buckling of Eccentrically Stiffened Orthotropic Cylinders, NASA TN D-4283, Jan. 1968.

TABLE 1a - DIMENSIONS AND CRITICAL LOADS OF SIMPLY SUPPORTED RING STIFFENED-SHELLS FOR DIFFERENT IN-PLANE BOUNDARY CONDITIONS $(e_2/h) = \pm 1$

$A_2/ah = .5; \quad e_2/h = 1; \quad I_{22}/ah^3 = 2; \quad \eta_{t2} = 3$

L/R	R/h	Z	SS1						SS2						SS3						SS4					
			+			-			+			-			+			-			+			-		
			λ	n	t	λ	n	t	λ	n	t	λ	n	t	λ	n	t	λ	n	t	λ	n	t	λ	n	t
.03	100	.0859	22000	3	0	22000	3	0	22000	1	0	22000	1	0	22000	3	0	22000	3	0	22000	1	0	22000	1	0
	500	.429	22100	1	2	22100	1	2	22100	1	2	22100	1	2	22700	3	0	22700	3	0	22700	1	0	22700	1	0
	1000	.859	22600	3	2	22600	3	2	22600	3	2	22600	3	2	24900	1	0	24900	1	0	25100	1	0	25100	1	0
	2000	1.72	24200	3	2	24300	3	2	24200	3	2	24300	3	2	33900	3	0	33900	3	0	34500	3	0	34500	1	0
.05	100	.238	7950	1	2	7950	1	2	7950	1	2	7950	1	2	7980	1	0	7980	1	0	7980	1	0	7980	1	0
	500	1.19	8330	3	2	8330	3	2	8330	3	2	8330	3	2	9970	1	0	9970	1	0	10100	1	0	10100	1	0
	1000	2.38	9490	3	2	9500	3	2	9490	3	2	9500	3	2	16100	1	7	16200	1	0	16500	1	9	16600	1	0
	2000	4.77	14100	5	2	14100	5	2	14100	5	2	14100	5	2	35600	1	20	39900	2	0	35800	1	21	39900	2	0
.1	100	.954	2080	3	2	2080	3	2	2080	3	2	2080	3	2	2310		0	2310	3	0	2320	1	0	2320	3	0
	500	4.77	3540	5	2	3550	5	2	3540	5	2	3550	5	2	8890	1	10	9970	2	0	8970	1	10	9970	2	0
	1000	9.54	7590	5	2	7610	5	2	7590	5	2	7610	5	2	16100	2	7	16200	2	0	16100	2	7	16200	2	0
	2000	19.1	16700	7	2	16700	7	2	16700	7	2	16700	7	2	32500	3	4	32500	3	0	32600	3	5	32600	3	0
.15	100	2.15	1060	3	2	1060	3	2	1060	3	2	1060	3	2	1620	1	0	1620	1	0	1660	3	2	1660	1	0
	500	10.7	3930	5	2	3950	5	2	3930	5	2	3950	5	2	8070	4	7	8180	4	0	8070	2	7	8180	4	0
	1000	21.5	8270	7	2	8290	7	2	8270	7	2	8290	7	2	16100	5	7	16200	5	0	16200	3	7	16200	3	0
	2000	42.9	16200	9	2	16200	9	2	16200	9	2	16200	9	2	32300	6	13	32700	6	0	32300	4	13	32700	6	0

TABLE 1a - CONTINUED.

L/R	R/h	Z	SS1						SS2						SS3						SS4					
			+			-			+			-			+			-			+			-		
			λ	n	t	λ	n	t	λ	n	t	λ	n	t	λ	n	t	λ	n	t	λ	n	t	λ	n	t
	100	3.82	780	3	2	790	3	2	780	3	2	790	3	2	1690	1	4	1820	1	0	1710	1	4	1879	1	2
	500	19.08	4180	7	2	4200	7	2	4180	7	2	4200	7	2	8130	3	2	8130	3	0	8150	[illegible]	0	8150	3	0
.2	1000	38.2	8090	9	2	8110	9	2	8090	9	2	8110	9	2	16100	6	7	16200	4	0	16100	4	7	16200	4	0
	2000	76.3	16300	11	2	16300	11	2	16300	11	2	16300	11	2	32500	6	4	32500	6	0	32500	6	0	32500	6	0
	100	5.96	720	5	2	740	5	2	720	5	2	740	5	2	1780	2	0	1790	2	0	1780	2	0	1780	2	0
.25	500	29.8	4070	7	2	4090	7	2	4070	7	2	4090	7	2	8230	5	8	8300	6	0	8240	3	9	8300	6	0
	1000	59.6	8160	9	2	8160	11	2	8160	9	2	8160	11	2	16100	7	7	16200	7	0	16200	5	7	16200	5	0
	2000	119	16300	13	2	16300	13	2	16300	13	2	16300	13	2	32200	9	11	32200	7	0	32300	7	11	32500	7	0
	100	8.59	760	5	2	783	5	2	760	5	2	783	5	2	1620	2	0	1620	2	0	1620	2	0	1620	2	0
	500	42.9	4070	9	2	4090	9	2	4070	9	2	4090	9	2	8070	4	7	8180	4	0	8070	4	7	8180	4	0
.3	1000	85.9	8150	11	2	8170	11	2	8150	11	2	8170	11	2	16100	6	7	16200	6	0	16100	6	7	16200	8	0
	2000	172	16300	15	2	16300	15	2	16300	15	2	16300	15	2	32300	8	13	32500	9	0	32300	8	13	32600	11	0
	100	23.8	848	7	2	870	7	2	848	7	2	870	7	2	1610	3	3	1630	3	0	1620	3	3	1640	3	0
	500	119	4090	13	2	4110	13	2	4090	13	2	4110	13	2	8060	7	5	8100	7	0	8070	7	6	8110	7	0
.5	1000	238	8180	17	2	8200	17	2	8180	17	2	8200	17	2	16100	10	7	16200	10	0	16200	10	7	16200	10	0
	2000	477	16400	23	2	16400	23	2	16400	23	2	16400	23	2	32200	14	11	32400	14	0	32400	14	0	32400	14	0
	100	95.4	853	11	2	873	11	2	850	11	2	875	11	2	1610	6	3	1630	6	0	1610	6	3	1630	6	0
	500	477	4120	23	2	4140	23	2	4120	23	2	4140	23	2	8060	14	5	8100	14	0	8100	14	0	8100	14	0
1	1000	954	8300	29	2	8320	29	2	8300	29	2	8320	29	2	16100	19	9	16200	20	0	16200	20	0	16200	20	0
	2000	1908	16600	41	2	16700	39	2	16700	39	2	16700	39	2	32200	28	11	32400	29	0	32400	28	0	32400	28	0
	100	382	876	21	2	900	21	2	876	21	2	900	21	2	1610	12	3	1620	13	0	1620	12	3	1630	12	0
	500	1908	4200	39	2	4230	39	2	4200	39	2	4230	39	2	8060	28	5	8100	29	0	8100	28	0	8100	28	0
2	1000	3820	8500	51	2	8530	51	2	8500	51	2	8530	51	2	16100	38	9	16200	41	0	16200	40	0	16200	40	0
	2000	7630	17200	69	2	17200	69	2	17200	69	2	17200	69	2	32400	56	0	32400	56	0	32400	56	0	32400	56	0

TABLE 1b - DIMENSIONS AND CRITICAL LOADS OF SIMPLY SUPPORTED RING-STIFFENED SHELLS FOR DIFFERENT IN-PLANE BOUNDARY CONDITIONS, $(e_2/h) = \pm 5$.

$A_2/ah = .5$; $e_2/h = 5$; $I_{22}/ah^3 = 2$; $\eta_{t2} = 3$

L/R	R/h	Z	SS1						SS2						SS3						SS4					
			+			-			+			-			+			-			+			-		
			λ	n	t	λ	n	t	λ	n	t		n	t	λ	n	t	λ	n	t		n	t	λ	n	t
	100	.0859	22000	3	0	22000	3	0	22000	3	0	22000	1	0	22000	3	0	22000	3	0	22000	1	0	22000	1	0
	500	.429	22100	1	2	22100	1	2	22100	1	2	22100	1	2	22700	-	0	22700	3	0	22700	1	0	22700	1	0
.03	1000	.859	22600	3	2	22600	3	2	22600	3	2	22600	3	2	25000	1	4	25000	3	0	25100	1	4	25100	1	0
	2000	1.72	24200	3	2	24300	3	2	24200	3	2	24300	3	2	32600	3	14	33900	3	0	33100	3	14	34500	3	0
	100	.238	7950	1	2	7950	1	2	7950	3	2	7950	1	2	7980	1	0	7980	1	0	7980	1	0	7980	1	0
	500	1.19	8320	3	2	8340	3	2	8320	3	2	8340	3	2	9890	3	5	9970	3	0	9990	1	6	10100	1	0
.05	1000	2.38	9480	3	2	9520	3	2	9480	3	2	9520	3	2	14800	3	11	16200	3	0	15100	1	11	16600	1	0
	2000	4.77	14000	5	3	14100	5	2	14000	5	3	14100	5	2	31100	3	17	39900	4	0	31800	1	17	39900	4	0
	100	.954	2070	3	2	2090	3	2	2070	3	2	2090	3	2	2300	1	2	2300	3	0	2320	1	2	2320	1	0
	500	4.77	3500	3	2	3590	5	2	3500	3	2	3590	5	2	7800	1	8	9970	2	0	7980	1	9	9970	2	0
.1	1000	9.54	7480	5	4	7670	5	2	7480	5	4	7670	5	2	14800	4	11	16200	2	0	14800	2	11	16200	2	0
	2000	19.1	16600	7	5	16700	7	2	16600	7	5	16700	7	2	30300	3	15	32500	3	0	30400	3	15	32600	3	0
	100	2.15	1040	3	2	1030	3	2	1040	3	2	1080	3	2	1510	3	3	1620	3	0	1540	1	3	1660	1	0
	500	10.7	3860	5	3	4010	5	2	3860	5	3	4010	5	2	7270	2	8	8180	4	0	7270	2	8	8180	4	0
.15	1000	21.5	8220	7	3	8330	7	2	8220	7	3	8330	7	2	14800	3	11	16200	5	0	14800	3	11	16240	3	0
	2000	42.9	16100	9	4	16300	9	2	16100	9	4	16300	9	2	29100	6	16	32700	6	0	29100	4	16	32700	6	0
	100	3.82	760	3	2	826	3	2	760	3	2	826	3	2	1480	1	4	1820	1	0	1520	1	4	1890	1	0
	500	19.08	4140	7	2	4250	7	2	4140	7	2	4250	7	2	7590	3	7	8130	3	0	7600	3	7	8150	3	0
.2	1000	38.2	8040	9	3	8160	9	2	8040	9	3	8160	9	2	14800	4	11	16200	4	0	14800	6	11	16200	4	0
	2000	76.3	16200	11	4	16300	11	2	16200	11	4	16300	11	2	28900	7	16	32500	6	0	29000	7	16	32500	6	0

TABLE 1b - CONTINUED.

L/R	R/h	Z	SS1						SS2						SS3						SS4					
			+			-			+			-			+			-			+			-		
			λ	n	t	λ	n	t	λ	n	t	λ	n	t	λ	n	t	λ	n	t	λ	n	t	λ	n	t
	100	5.96	704	5	2	797	5	2	704	5	2	798	5	2	1690	3	4	1780	2	0	1720	1	4	1780	2	0
	500	29.8	4030	7	2	4140	7	2	4030	7	2	4140	7	2	7260	5	8	8300	6	0	7260	3	8	8300	6	0
.25	1000	59.6	8120	9	3	8200	11	2	8120	9	3	8200	11	2	14700	6	12	16200	7	0	14700	4	12	16200	5	0
	2000	119	16200	13	3	16400	13	2	16200	13	3	16400	13	2	29000	8	16	32400	9	0	29100	6	16	32500	7	0
	100	8.59	730	5	2	856	5	2	740	5	2	858	5	2	1520	4	3	1620	2	0	1520	2	3	1620	2	0
	500	42.9	4030	9	2	4140	9	2	4030	9	2	4140	9	2	7270	4	8	8160	4	0	7280	4	8	8180	4	0
.3	1000	85.9	8110	11	2	8220	11	2	8110	11	2	8220	11	2	14600	5	12	16200	6	0	14600	5	11	16200	6	0
	2000	172	16300	15	3	16400	15	2	16300	15	3	16400	15	2	29100	8	16	32500	9	0	29200	10	16	32600	11	0
	100	23.8	850	7	2	940	7	2	850	7	2	950	7	2	1470	3	3	1630	3	0	1470	3	3	1640	3	0
	500	119	4060	13	2	4170	13	2	4060	13	2	4170	13	2	7260	6	8	8100	7	0	7270	6	8	8110	7	0
.5	1000	238	8150	17	2	8260	17	2	8150	17	2	8260	17	2	14500	9	11	16200	10	0	14500	9	11	16200	10	0
	2000	477	16400	23	2	16500	23	2	16400	23	2	16500	23	2	29000	13	16	32400	14	0	29100	12	16	32400	14	0
	100	95.4	880	11	2	980	11	2	880	11	2	990	11	2	1470	6	3	1630	6	0	1470	6	3	1630	6	0
	115	109.7	998	13	2										1670	6	4									
1	500	477	4100	23	2	4220	23	2	4100	23	2	4220	23	2	7240	13	6	8100	14	0	7280	12	8	8100	14	0
	1000	954	8270	29	2	8390	29	2	8270	29	2	8390	29	2	14500	18	11	16200	22	0	14600	18	11	16200	20	0
	2000	1910	16700	39	2	16800	39	2	16700	39	2	16800	39	2	28900	25	16	32400	29	0	29200	26	16	32400	28	0
	100	382	900	20	2	1010	20	2	960	21	2	1070	21	2	1460	11	4	1620	13	0	1460	11	4	1630	15	0
	500	1910	4220	39	2	4330	39	2	4220	39	2	4340	39	2	7230	25	8	8100	29	0	7300	25	8	8100	28	0
2	1000	3820	8500	51	2	8620	51	2	8500	51	2	8630	51	2	14500	36	11	16200	40	0	14600	36	11	16200	40	0
	2000	7630	17200	69	2	17200	69	2	17200	69	2	17300	69	2	29000	49	16	32400	56	0	29300	49	16	32400	56	0

TABLE 2 - DIMENSIONS AND CRITICAL LOADS FOR STUDY OF EFFECT OF INCREASE OF STIFFENER ECCENTRICITY

$A_2/ah = .5$; $R/h = 1000$; $I_{22}/ah^3 = 5$; $n_{t2} = 3$

L/R	e_2/h	SS1						SS2						SS3						SS4					
		+			-			+			-			+			-			+			-		
		λ	n	t	λ	n	t	λ	n	t	λ	n	t	λ	n	t	λ	n	t	λ	n	t	λ	n	t
	1	8300	29	2	8330	29	2	8300	29	2	8330	29	2	16200	20	0	16200	20	0	16200	20	0	16200	20	0
	3	8290	29	2	8360	29	2	8290	29	2	8360	29	2	15400	19	10	16200	20	0	15600	19	10	16200	20	0
1	5	8280	29	2	8400	29	2	8280	29	2	8400	29	2	14800	18	11	16200	20	0	14900	18	10	16200	20	0
	8	8260	29	2	8470	29	2	8290	29	2	8470	29	2	14200	18	9	16200	20	0	14300	18	9	16200	20	0
	10	8290	29	2	8520	29	2	8300	29	2	8530	29	2	14000	20	9	16200	20	0	14100	19	9	16200	20	0
	1	8730	71	2	8750	71	2	8730	71	2	8750	71	2	16200	57	7	16200	59	0	16300	58	0	16300	58	0
	3	8720	71	2	8800	71	2	8720	71	2	8800	71	2	15500	54	11	16200	59	0	15700	53	11	16300	58	0
3	5	8720	72	2	8850	72	2	8740	71	2	8870	71	[illegible]	14800	53	11	16200	59	0	15000	52	11	16300	58	0
	8	8710	72	2	8920	72	2	8800	71	2	9020	71	2	14300	52	10	16200	59	0	14400	52	10	16300	58	0
	10	8870	71	2	8980	72	2	8870	71	2	9140	71	2	14000	53	9	16200	59	0	14200	52	9	16300	58	0

TABLE 5 - DIMENSIONS AND CRITICAL LOADS FOR INVESTIGATION OF EFFECT OF INCREASE OF STIFFENER AREA

$R/h = 1000$; $e_2/h = \pm 5$; $I_{22}ah^3 = 5$; $n_{t2} = 3$

L/R	A_2/ah	SS1						SS2						SS3						SS4					
		$e_2/h = 5$			$e_2/h = -5$			$e_2/h = 5$			$e_2/h = -5$			$e_2/h = 5$			$e_2/h = -5$			$e_2/h = 5$			$e_2/h = -5$		
		λ	n	t	λ	n	t	λ	n	t	λ	n	t	λ	n	t	λ	n	t	λ	n	t	λ	n	t
1	.05	6950	27	2	6970	27	2	6950	27	2	6970	27	2	13600	18	2	13600	19	0	13600	18	0	13600	18	0
	.1	7120	27	2	7150	27	2	7120	27	2	7150	27	2	13900	19	2	13900	19	0	13900	18	0	13900	18	0
	.5	8280	29	2	8400	29	2	8280	29	2	8400	29	2	14800	18	11	16200	20	0	14900	18	10	16200	20	0
	1	9530	31	2	9730	31	2	9530	31	2	9730	31	2	15000	18	12	18700	22	0	15100	18	12	18700	22	0
	1.5	10600	33	2	10900	33	2	10600	33	2	10900	33	2	15100	18	12	20900	23	0	15200	18	12	21000	24	0
	5	14900	21	12	17000	39	2	15100	21	12	17000	39	2	15300	18	13	32400	29	0	15400	18	13	32400	28	0
3	.1	7460	67	2	7490	67	2	7460	67	2	7490	67	2	13900	55	0	13900	55	0	13900	54	0	13900	54	0
	.5	8710	72	2	8850	72	2	8740	71	2	8870	71	2	14800	53	11	16200	59	0	15000	52	11	16300	58	0
	1	10100	76	2	10200	77	2	10100	75	2	10300	77	2	15100	52	12	18700	63	0	15200	52	12	18800	62	0
	1.5	11300	79	2	11600	80	2	14600	56	11	11600	79	2	15100	52	12	21000	66	0	15300	52	12	21000	66	0

TABLE 4 - EFFECT OF INCREASED MOMENT OF INERTIA OF STIFFENER ON THE CRITICAL LOADS.

$A_2/ah = .5$; $R/h = 1000$; $I_{22}/ah^3 = 5$; $n_{t2} = 3$

L/R	e_2/h	SS1						SS2						SS3						SS4					
		+			-			+			-			+			-			+			-		
		λ	n	t	λ	n	t	λ	n	t	λ	n	t	λ	n	t	λ	n	t	λ	n	t	λ	n	t
1	± 1	8300	29	2	8330	29	2	8300	29	2	8330	29	2	16200	20	5	16200	20	0	16200	20	0	16200	20	0
	± 5	8280	29	2	8400	29	2	8280	29	2	8400	29	2	14800	18	11	16200	20	0	14900	18	10	16200	20	0
3	± 1	8730	71	2	8750	71	2	8730	71	2	8750	71	2	16200	57	7	16200	59	0	16300	58	0	16300	58	0
	± 5	8720	72	2	8850	72	2	8740	71	2	8870	71	2	14800	51	12	16200	59	0	15000	52	11	16300	58	0

TABLE 5 - DIMENSIONS AND CRITICAL LOADS FOR STUDY OF EFFECT OF INCREASE OF STIFFENER MOMENT OF INERTIA

$R/h = 1000$; $A_2/ah = .5$; $e_2/h = \pm 5$; $n_{t2} = 3$

l/R	I_{22}/ah^3	SS1						SS2						SS3						SS4					
		+			-			+			-			+			-			+			-		
		λ	n	t	λ	n	t	λ	n	t	λ	n	t	λ	n	t	λ	n	t	λ	n	t	λ	n	t
1	1	8270	29	2	8390	29	2	8270	29	2	8390	29	2	14400	18	12	16200	20	10	14500	17	12	16200	20	0
	2	8270	29	2	8390	29	2	8270	29	2	8390	29	2	14500	18	11	16200	20	0	14600	18	11	16200	20	0
	3	8280	29	2	8390	29	2	8280	29	2	8390	29	2	14600	20	11	16200	20	0	14700	18	11	16200	20	0
	5	8280	29	2	8400	29	2	8280	29	2	8400	29	2	14800	18	11	16200	20	0	14900	18	10	16200	20	0
	8	8290	29	2	8400	29	2	8290	29	2	8400	29	2	15000	18	10	16200	20	0	15100	18	10	16200	20	0
	10	8300	29	2	8410	29	2	8300	29	2	8410	29	2	15100	19	9	16200	20	0	15200	19	9	16200	20	0
	50	8410	29	2	8520	29	2	8410	29	2	8520	29	2	15800	20	6	16200	20	0	15500	20	6	16200	20	0
3	1	8710	72	2	8840	72	2	8710	71	2	8840	71	2	14400	51	12	16200	59	0	14500	51	12	16300	58	0
	2	8710	72	2	8840	72	2	8720	71	2	8850	71	2	14500	51	12	16200	59	0	14700	51	12	16300	58	0
	3	8710	72	2	8840	72	2	8720	71	2	8860	71	2	14600	52	11	16200	59	0	14800	52	11	16300	58	0
	5	8720	72	2	8850	72	2	8740	71	2	8870	71	2	14800	53	11	16200	59	0	15000	52	11	16300	58	0
	8	8720	72	2	8860	72	2	8760	71	2	8900	71	2	15000	54	10	16200	59	0	15200	53	10	16300	58	0

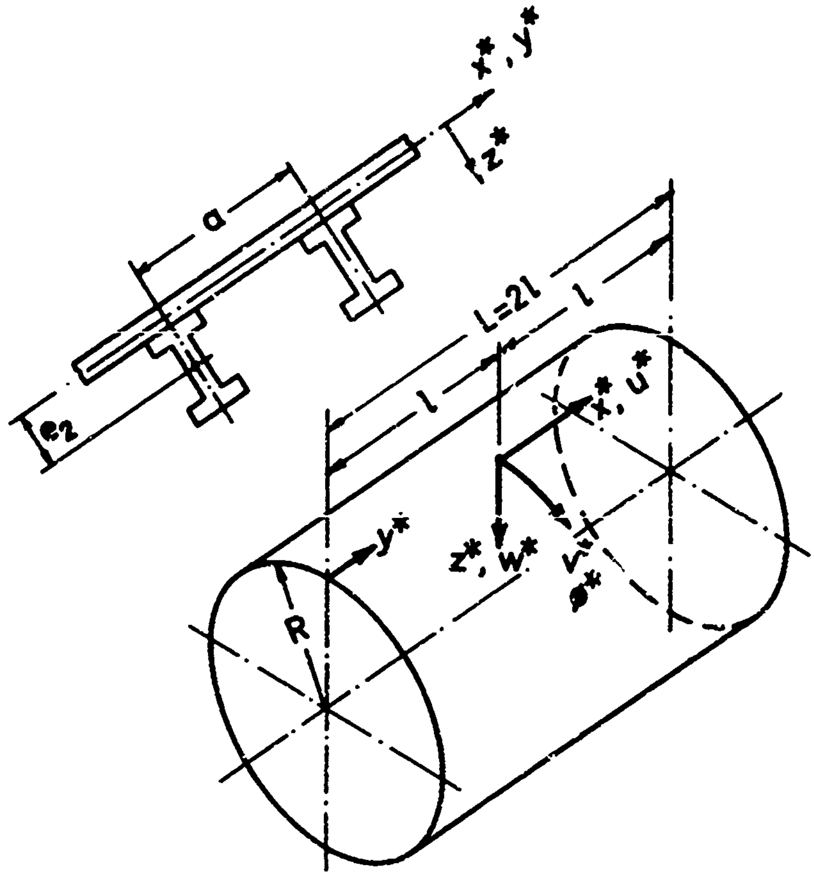


FIG. 1 NOTATION

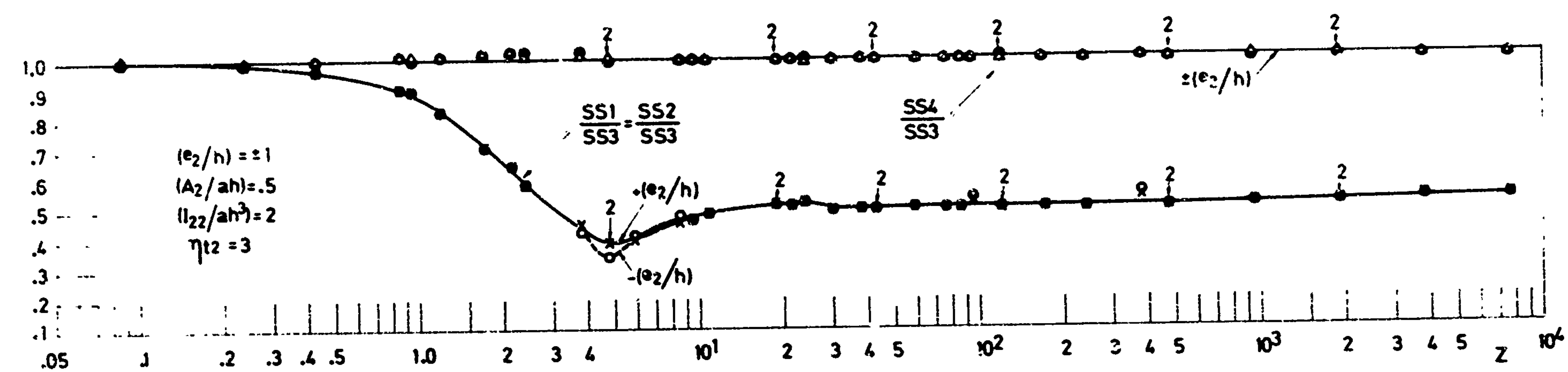


FIG. 2a VARIATION OF RATIO OF BUCKLING LOADS WITH SHELL GEOMETRY, $(e_2/h) = \pm 1$

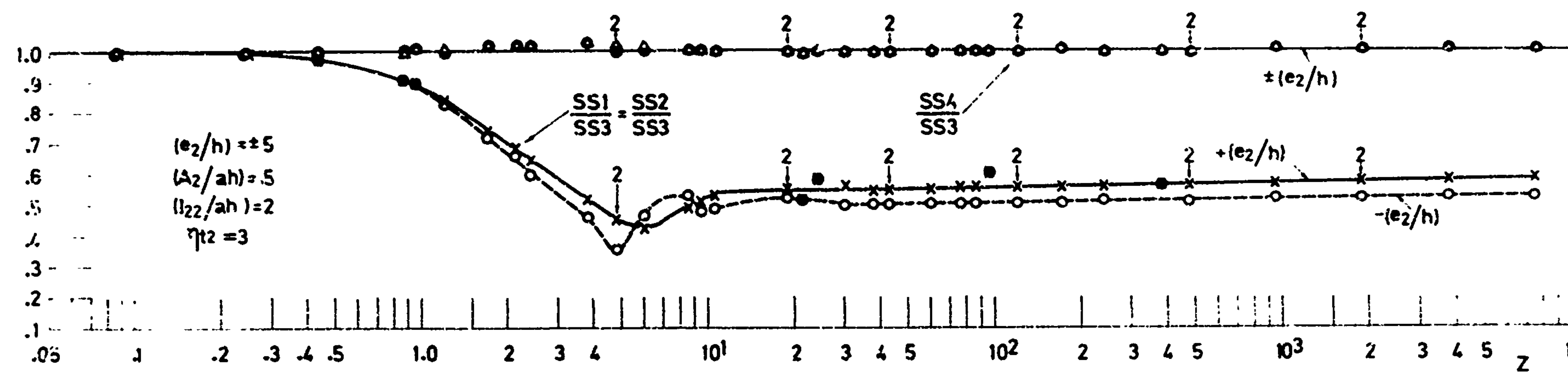


FIG. 2b VARIATION OF RATIO OF BUCKLING LOADS WITH SHELL GEOMETRY, $(e_2/h) = \pm 5$

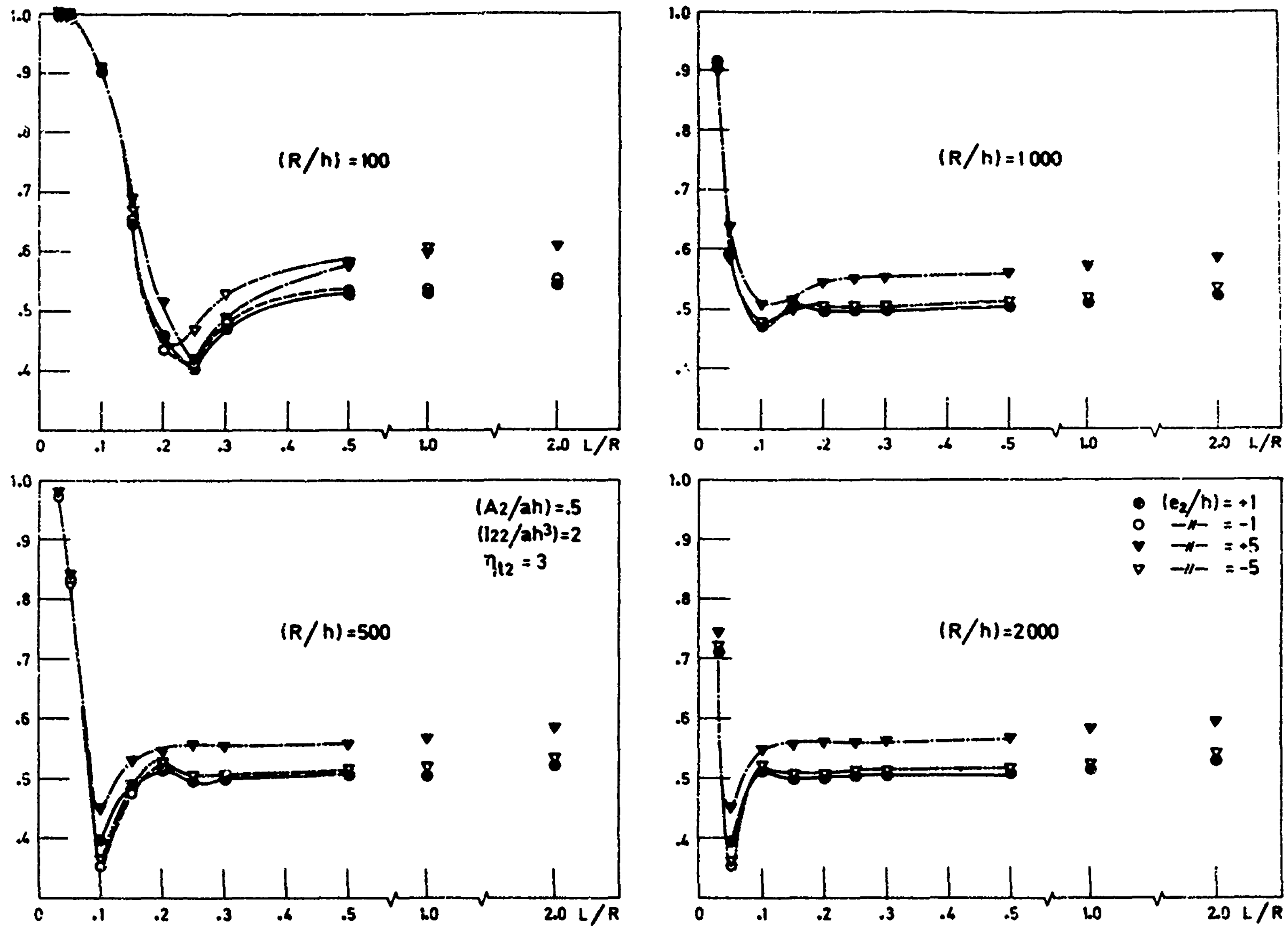
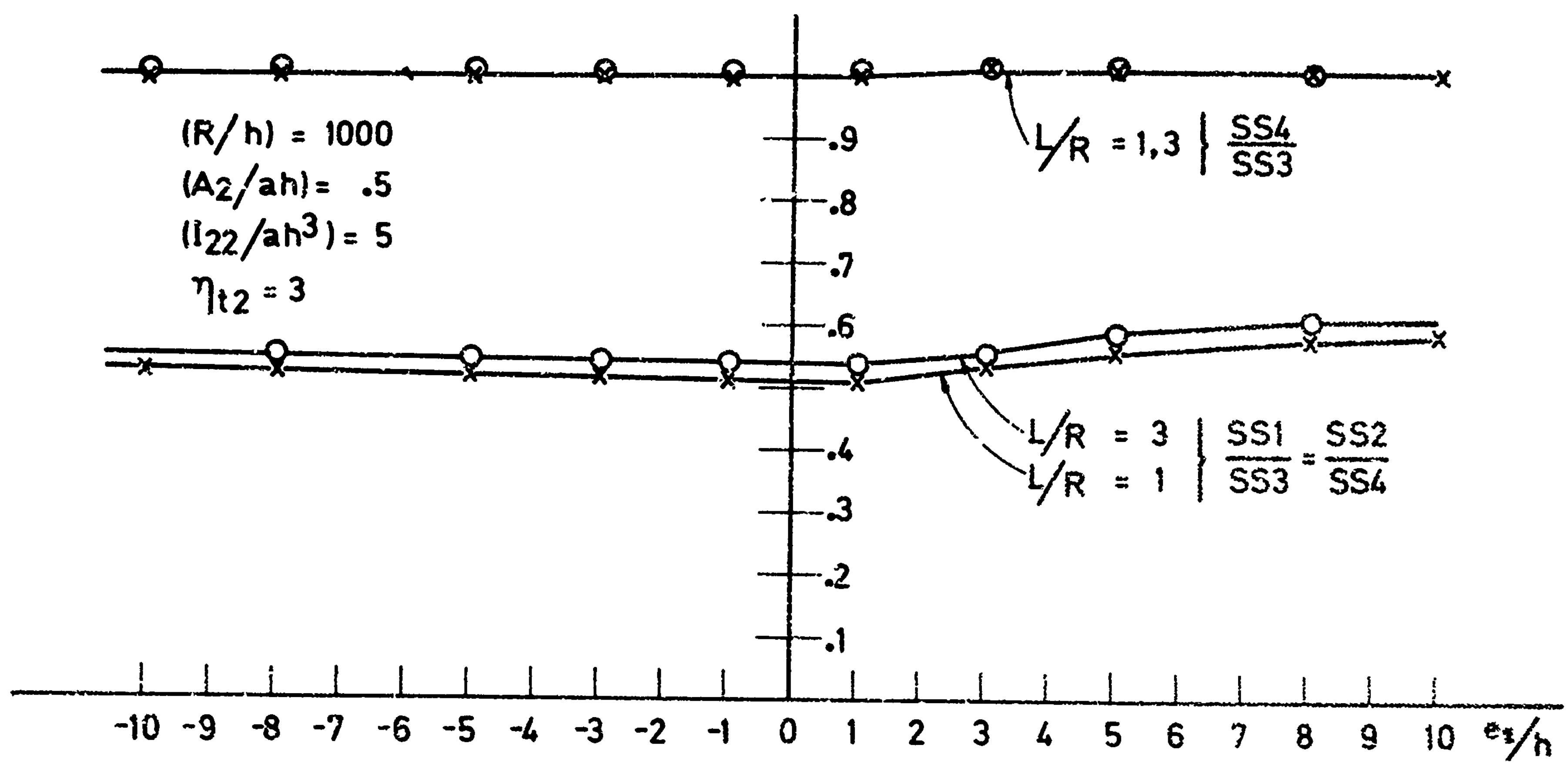
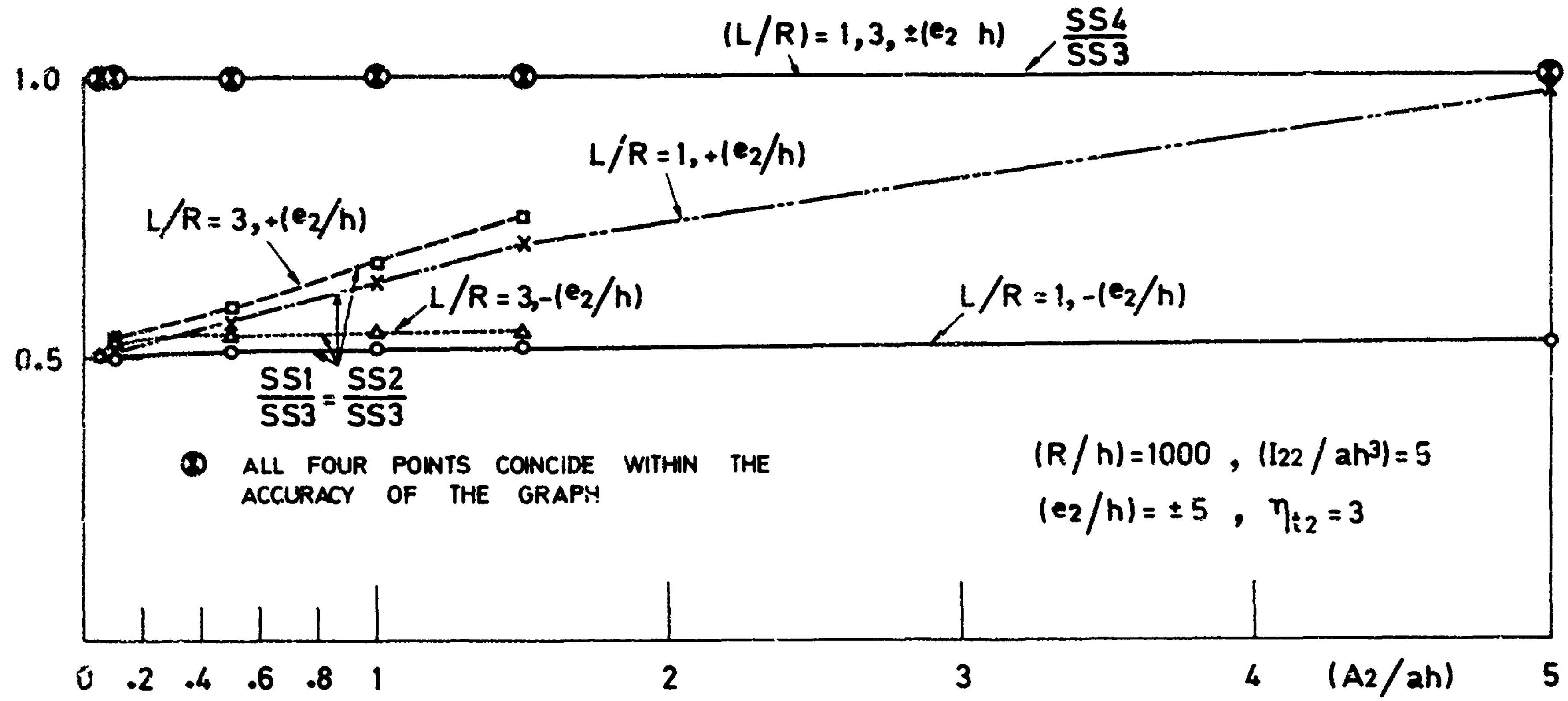


FIG. 3 VARIATION OF RATIO OF BUCKLING LOADS WITH SHELL LENGTH FOR DIFFERENT (R/h) RATIOS



<u>FIG. 4</u> EFFECT OF STIFFENER ECCENTRICITY ON THE RATIO OF BUCKLING LOADS



<u>FIG. 5</u> EFFECT OF STIFFENER AREA ON THE RATIO OF BUCKLING LOADS

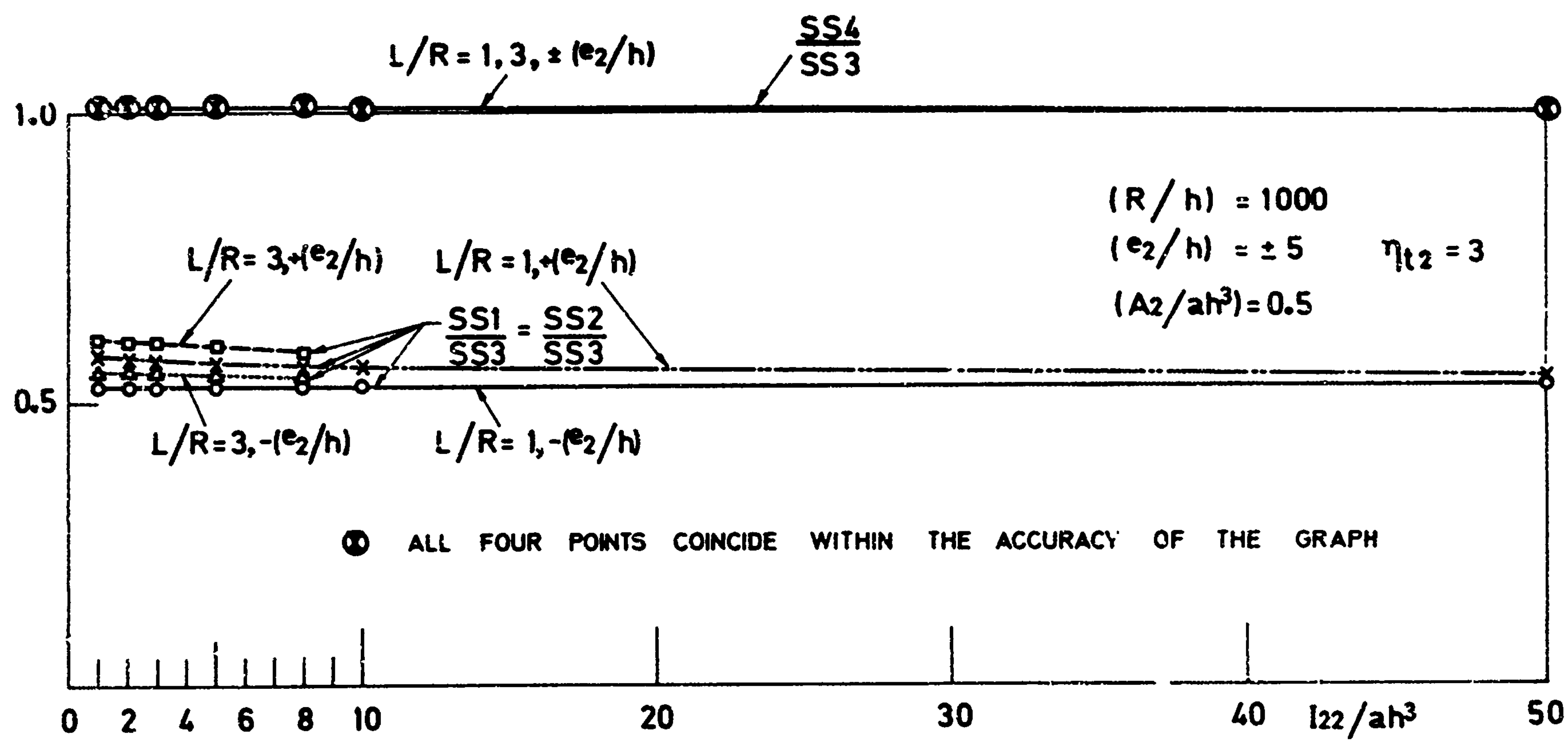


FIG. 6 EFFECT OF STIFFENER MOMENT OF INERTIA ON THE RATIO OF BUCKLING LOADS

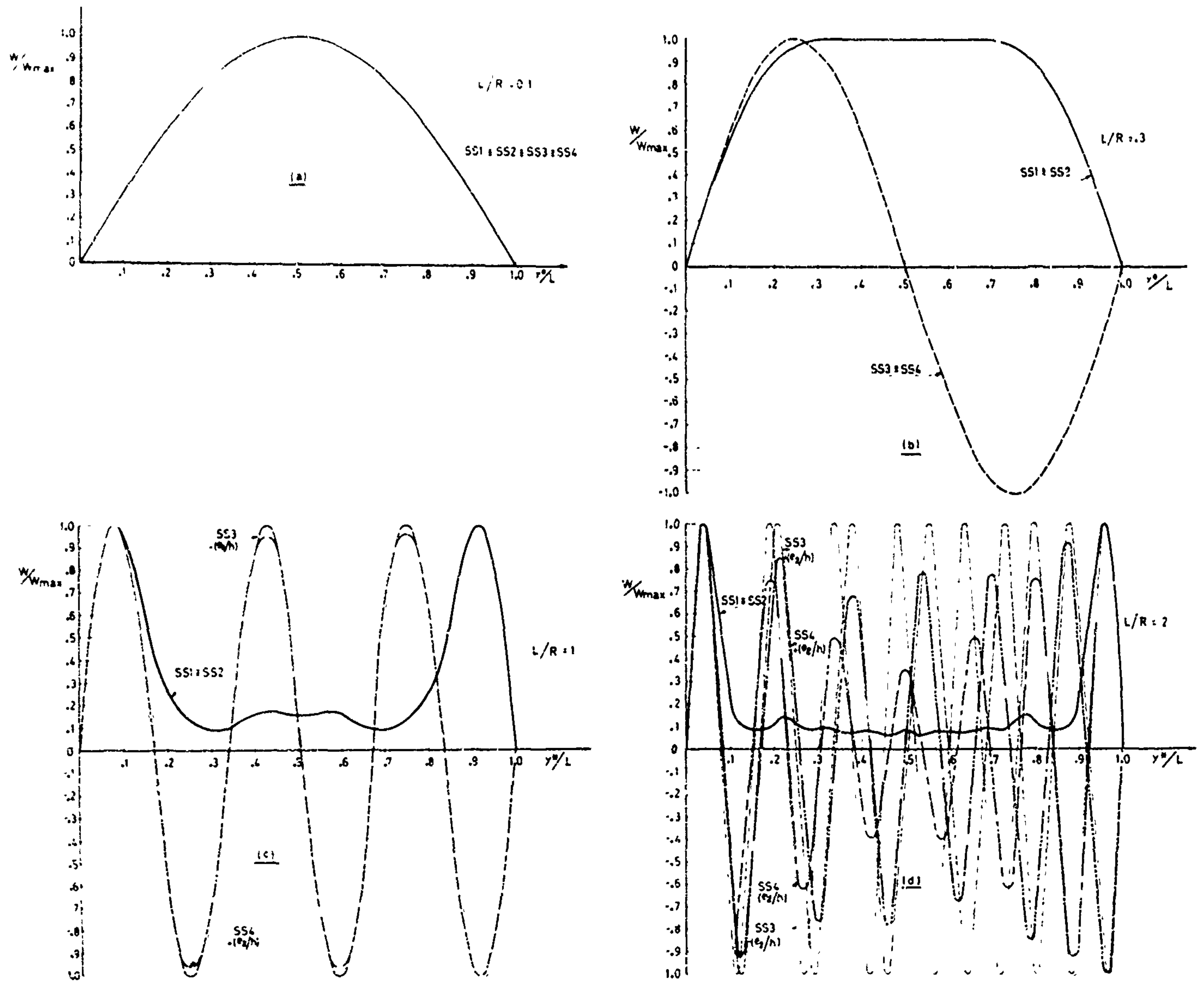


FIG. 7 BUCKLING MODES OF "THICK" SHELLS (R/h) = 100.
WITH LOW ECCENTRICITY (e_2/h) = ±1, (A_2/ah) = 0.5, (I_{22}/ah^3) = 2

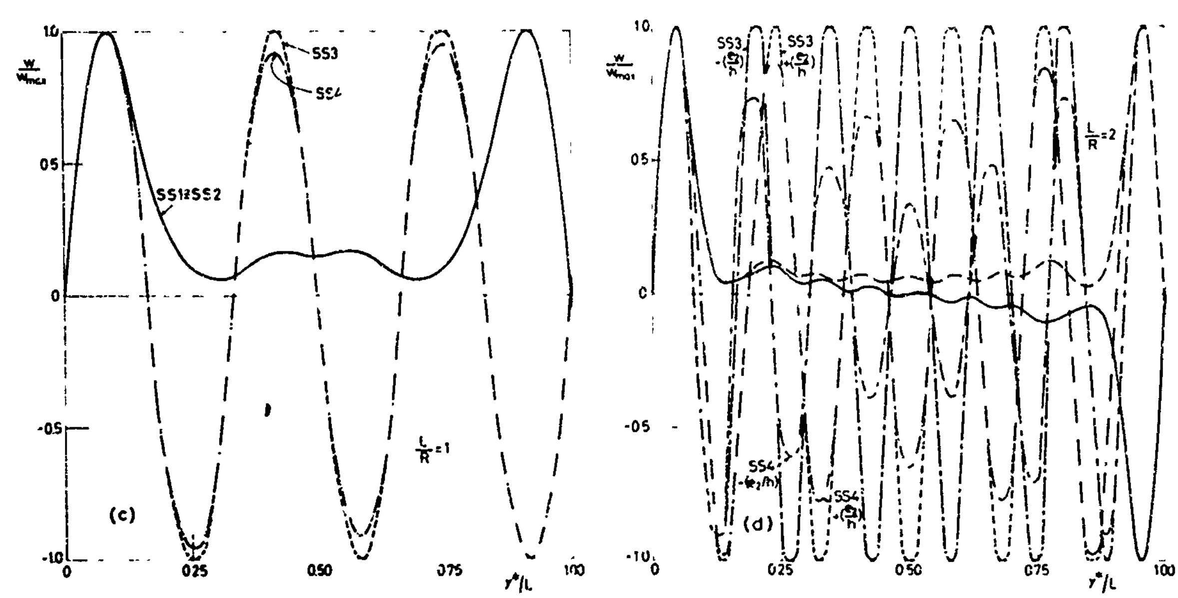


FIG 8 EFFECT OF STIFFENER ECCENTRICITY, $(e_2/h)=\pm 5$ ON THE BUCKLING MODES OF "THICK" SHELLS, $(R/h)=100$
$(A_2/ah)=0.5$, $(I_{22}/ah^3)=2$

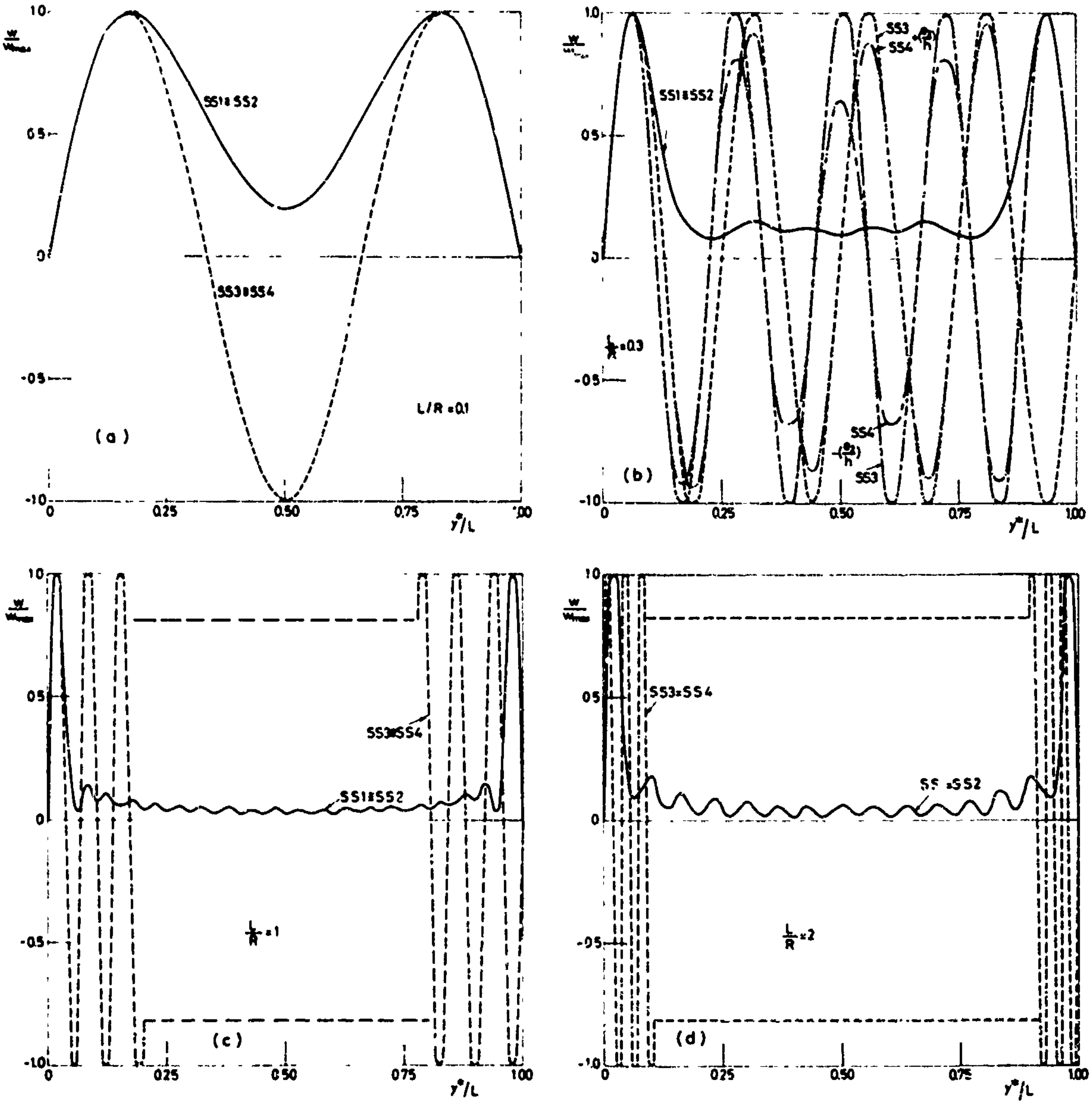


FIG. 9 BUCKLING MODES OF "THIN" SHELLS, $(R/h) = 2000$
$e_2/h = \pm 1$, $A_2/ah = 0.5$, $I_{22}/ah^3 = 2$

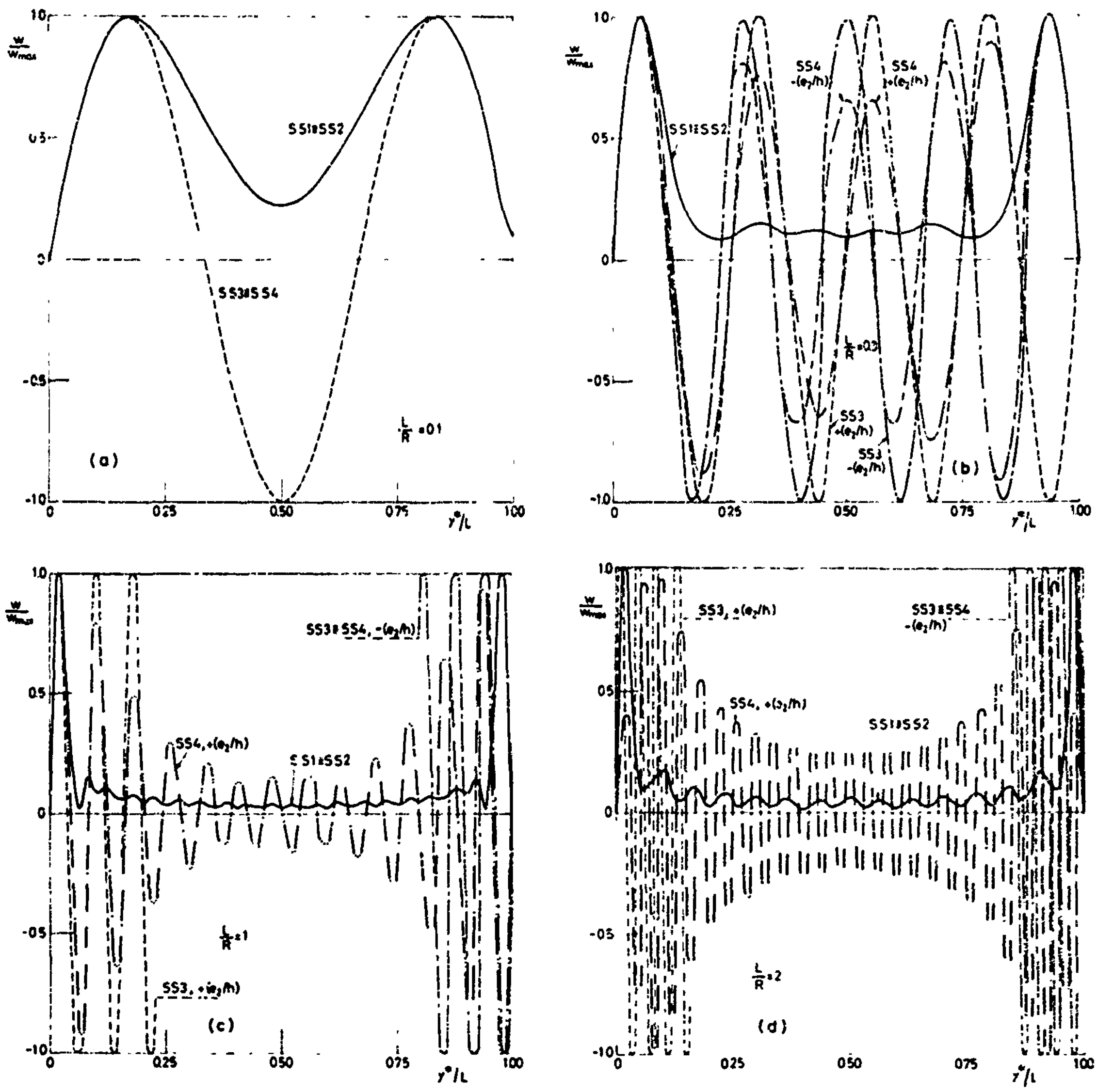


FIG 10 EFFECT OF STIFFENER ECCENTRICITY, $(e_2/h)=\pm 5$, ON THE BUCKLING MODES OF "THIN" SHELLS, $(R/h)=2000$

$A_2/ah = 05$, $\bar{I}_{22}/ah^3 = 2$

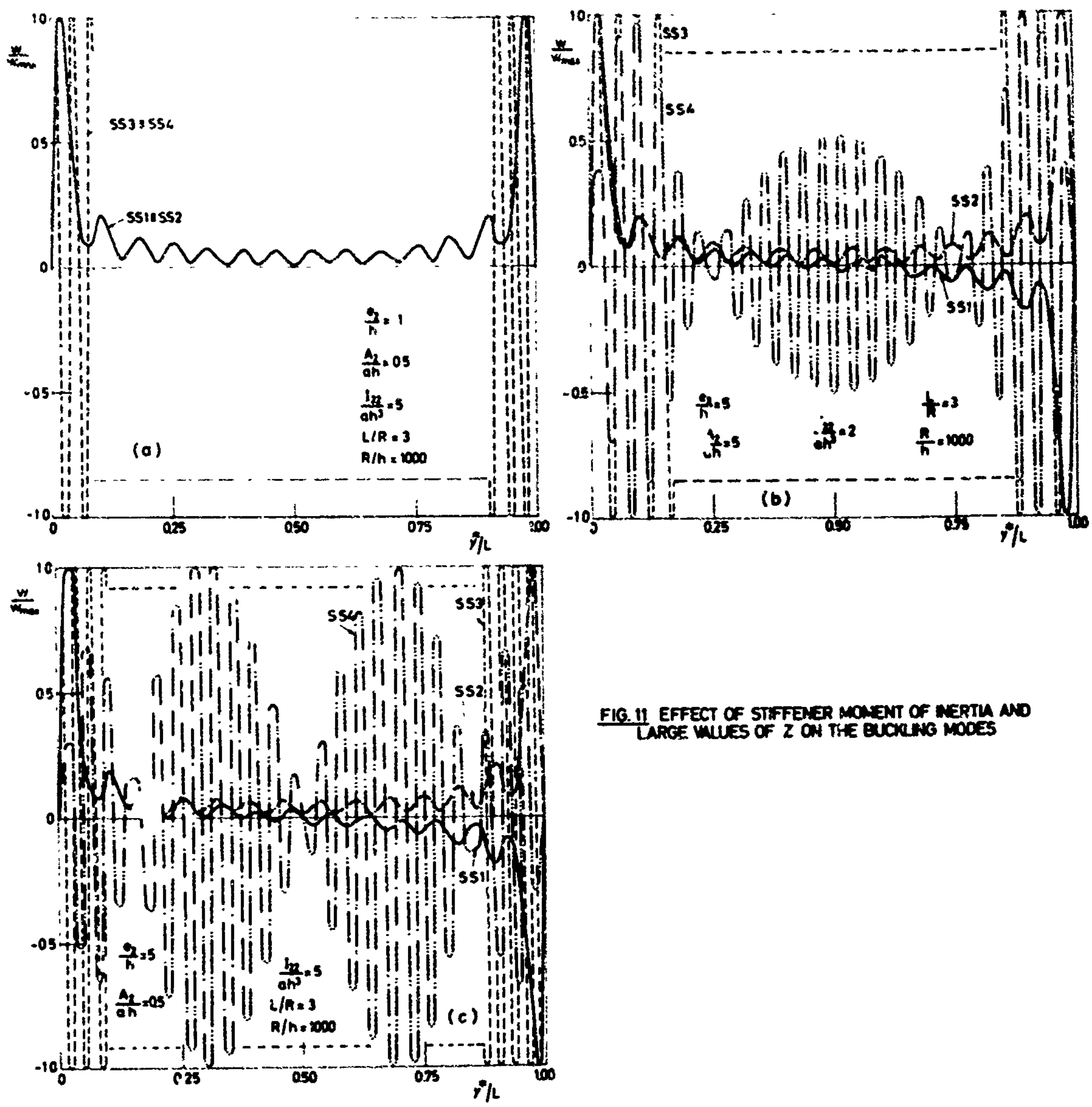


FIG. 11 EFFECT OF STIFFENER MOMENT OF INERTIA AND LARGE VALUES OF Z ON THE BUCKLING MODES

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)

TECHNION RESEARCH AND DEVELOPMENT FOUNDATION
DEPARTMENT OF AERONAUTICAL ENGINEERING
HAIFA, ISRAEL

2a. REPORT SECURITY CLASSIFICATION

UNCLASSIFIED

2b. GROUP

3. REPORT TITLE

INFLUENCE OF IN-PLANE BOUNDARY CONDITIONS ON BUCKLING OF RING-STIFFENED CYLINDRICAL SHELLS

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Scientific Interim

5. AUTHOR(S) (First name, middle initial, last name)

TANCHUM WELLER MENAHEM BARUCH JOSEF SINGER

6. REPORT DATE	7a. TOTAL NO OF PAGES	7b. NO OF REFS
October 1970	55	32

8a. CONTRACT OR GRANT NO

AF 61(052)-0905

b. PROJECT NO.

9782-01

c. 61102F

d. 681307

9a. ORIGINATOR'S REPORT NUMBER(S)

TAE REPORT No. 101

9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

AFOSR-TR- 71-[illegible]

10. DISTRIBUTION STATEMENT

1. This document has been approved for public release and sale; its distribution is unlimited.

11. SUPPLEMENTARY NOTES

TECH, OTHER

12. SPONSORING MILITARY ACTIVITY

AF Office of Scientific Research (NAM)
1400 Wilson Boulevard
Arlington, Virginia 22209

13. ABSTRACT

The effect of in-plane boundary conditions on the buckling loads of simply supported ring-stiffened cylindrical shells is studied. As in the case of unstiffened shells, the "weak" in-plane boundary conditions SS1 and SS2 yield here critical loads about one half of the "classical" loads. It was observed that the SS1 critical loads are identical with the SS2 loads and the SS4 loads are almost the same as the "classical" SS3 loads. The combined effect of stiffener parameters and in-plane boundary conditions is studied. For internally stiffened shells the influence of in-plane boundary conditions is found to diminish with increasing values of stiffener eccentricity and area. No such effect is observed for externally stiffened shells. The buckling modes are also studied and found that they are dependent upon shell length (or Z) and upon stiffener location and parameters.

DD FORM 1 NOV 65 1473

UNCLASSIFIED

Security Classification

UNCLASSIFIED
Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
STIFFENED CYLINDRICAL SHELLS						
INFLUENCE OF IN-PLANE BOUNDARY CONDITIONS						
INFLUENCE OF RINGS ON BOUNDARY CONDITION EFFECTS						

UNCLASSIFIED
Security Classification